TEXTILE MECHANICS

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Textile Mechanics

Textile Machine Drawing

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PREFACE

In the textile industry as a whole, there is an enormous number and variety of machines used; and since this variety is gradually being increased, and more elaborate mechanism introduced, the subject of Mechanics is becoming more and more important for those who are engaged in this industry. It is essential, then, that all who are employed in the mechanical production of yarns and cloth, and in the subsequent treatment of these products, should have, at least, an elementary knowledge of the principles underlying the construction of the machines and appliances used.

In certain respects, and in the elementary stages, the subject of Mechanics is possibly more difficult for the average student than those of Mathematics and Machine Drawing, both of which are dealt with in former publications of this series. The purely technological requirements of textile education are more or less fully recognized by all grades of technical schools in textile districts; but, in most cases, the textile student acquires what little scientific knowledge he possesses as he proceeds from stage to stage in his textile studies. The success attending this acquisition depends solely on the capability of the

student, and, as a result, much of the textile technologist's work is not appreciated at its full value.

The matter introduced in the following pages deals only with the main elementary principles. These are fundamental and of general application; but the work differs from other elementary treatises on Mechanics in that the examples chosen for demonstration, and the subsequent exercises, are arranged in language and terms to suit the present-day textile student. In this respect, the authors hope that their efforts to provide suitable elementary science books for those engaged in the textile industry will be appreciated.

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TEXTILE MECHANICS

CHAPTER I

MATTER, FORCE, ETC.

PRELIMINARY.—The science of **Mechanics** consists of two distinct parts: (*a*) **Statics**, which deals with forces not in motion; and (*b*) **Dynamics**, which deals with forces in motion. The subject matter of the latter part is, in general, of a much more complex nature than that of the former part; hence, in any elementary treatise, which may embody sections of both, statical questions should, naturally, be considered first. As a rule, practical men are not concerned much with the action of forces in the abstract, but are mainly interested in their technical and industrial applications. Force, or that which produces force, cannot be perceived by the senses, but the *effects* of force may be made evident in many ways. We shall endeavour to show how many kinds of force may be measured, and how these measurements may be utilized in practice. The principles involved, although illustrated where possible with special reference to textile problems, are of general application, and as such may be used for all kinds of mechanical work.

Matter.—In nearly every science special terms and phrases are found and used with advantage, and the definitions of these terms or phrases are often peculiar to a particular branch of knowledge. In addition, it is often found convenient to attach a particular and special meaning to some commonly-used word. Mechanics is no exception to these usages. The importance of being perfectly definite as to the meaning of the words, terms, and phrases used cannot be impressed too early or too emphatically on the beginner.

Mechanics is the study of force-actions, and force-actions are usually, although not invariably, manifested by their effect on the matter upon which they act. We must then, first of all, have some definite ideas as to the meaning of this word matter.

Matter may be defined as anything which can affect, or be perceived by, the senses, and which can be acted upon by force. It is often divided into two great classes, viz. solids and fluids; for our purpose, it will be more convenient to regard matter as being divided into three states or conditions, viz. solids, liquids, and gases.

Solids, such as wood, iron, ice, &c., tend always to retain their original shape and size. A comparatively great amount of force is necessary to cause them to change in these particulars. This, of course, does not apply to plastic substances, such as rubber, putty, and the like, all of which behave partly like solids and partly like liquids.

Liquids, e.g. alcohol, mercury, water, &c., are easily changed in shape, but it is very difficult to alter them in size. Gases—hydrogen, oxygen, coal-gas, steam, and the like—differ from both solids and liquids, since they are easily changed both in form and

dimensions, while they further possess the peculiar and important property of indefinite expansion.

FORCE.—Force is that which causes, or tends to

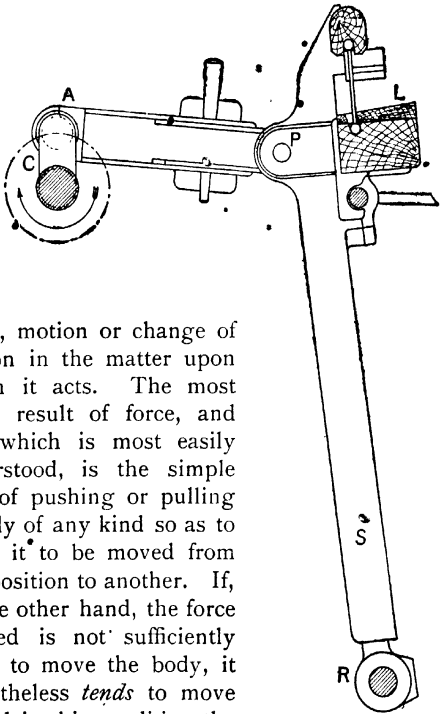

Fig. 1

cause, motion or change of motion in the matter upon which it acts. The most usual result of force, and that which is most easily understood, is the simple case of pushing or pulling a body of any kind so as to cause it to be moved from one position to another. If, on the other hand, the force applied is not sufficiently great to move the body, it nevertheless *tends* to move it, and in this condition the body may be subjected to one of many kinds of force-actions, e.g. pressure, tension, &c.

The first case is illustrated in fig. 1, where the turning force in the loom crank C—which may rotate counter-

clockwise, or clockwise as illustrated—is imparted to *the end A of the connecting arm. Due to the position* of the parts, the end P of the connecting arm receives an approximately rectilinear motion, and thus causes the pin P, and the lay or slay L, to move backwards and forwards. The swords S, which carry the lay, are fulcrumed on the rocking shaft R.

Fig. 2 shows the second case, where the downward force or pull of the weight W, by means of the lever L, the hanger H, and the strap S, is communicated to the pressing

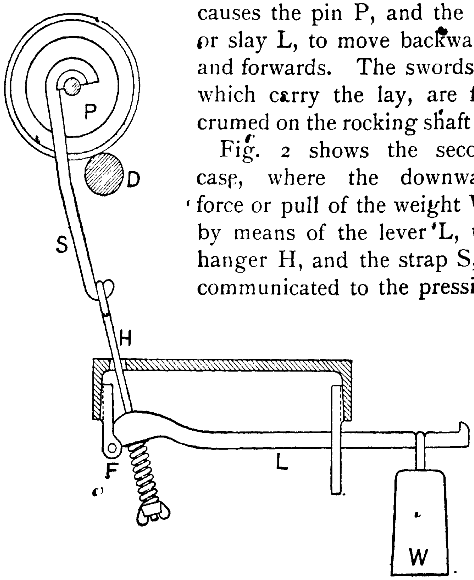

Fig. 2

roller P, and hence to the drawing, front, or boss roller D. In this case the weight or force W causes no motion, but it does produce pressure between the pressing roller P and the drawing roller D. Although causing no motion, the force due to weight W and the other parts nevertheless has a tendency to cause motion, as could be proved by taking away the drawing roller D, when the pressing

roller P would be immediately pulled forcibly downwards.

The consideration of force causing change of motion is more complex, since it introduces the question of acceleration. This phase can only be discussed at a later stage.

UNIT OF FORCE.—To enable definite problems to be solved, it is essential to have a standard unit for the measurement of force. The British unit of force is the pound avoirdupois, and is spoken of as the engineer's unit or gravitation unit.

A force of 1 lb. is that which will just support a weight of 1 lb. against the action of gravity.

The force of gravity, however, is not constant; it varies at different parts of the earth's surface. It follows that the force of 1 lb. is not constant. Nevertheless, the actual amount of variation is slight, and for all practical purposes the above definition can be taken as being exact. The British unit of force is absolutely correct only when measured at Greenwich sea-level.

FORCE ELEMENTS.—A force is completely defined only when the following three elements are known.

1. Its place or point of application.
2. The direction in which it acts.
3. Its magnitude or size.

A fourth element is sometimes added, viz. the sense, i.e. whether it is a push or a pull; in practice, however, this is known when elements 1 and 2 are known.

The point or place of application of a force may be actually a point, a line, or a surface. In the case of a surface, the force is generally considered as acting at the centre of the surface. In the case of a weight

acting as shown in fig. 2, the downward pull of the weight W is taken as acting at the centre of gravity of the weight. Fig. 2 also illustrates a case where the force is acting on a line, the line being the length of the drawing and pressing rollers, and its position that where the pressing roller P and the drawing roller D are in contact. In such cases, the pressure in pounds per inch of width is commonly adopted as a standard of comparison.

Fig. 3 is a simple example of a force being considered as acting at a point. A basket of cops B is

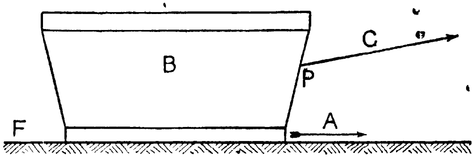

Fig. 3

being pulled along the floor F by means of a cord C. The pull in the cord is conveyed to the basket at the point P, which may be regarded as the point of application of the pulling force in the cord C.

The direction of a force is that line or path in which the force moves, or tends to move, the body on which it acts. In fig. 3, it will be noticed that the direction of the force is not the same as the direction in which the basket will move. The direction of the force is an oblique one, whereas the direction in which the basket will move is horizontal, along the floor in the direction indicated by the arrow A. Nevertheless, the basket will tend to move in the same direction as the force. Indeed, it will be seen later that only part of the force in the cord C is being used to pull the basket horizontally along the floor, while the

remainder of the force is tending to lift the basket from off the floor.

The magnitude or size of a force is the number of pounds of pull or push with which the force acts on a body. The pound is a convenient unit for many cases, but where the forces under consideration are large, any multiple unit of the pound, such as ton, hundredweight, &c., may be used.

Representation of Forces.—It is necessary for a complete understanding of certain problems, and to facilitate their solution, to be able to represent forces diagrammatically. They may be represented very

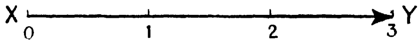

Fig. 4

conveniently by means of straight lines. Fig. 4 shows a straight line XY, 3 in. long. Either end of the line XY may be the point of application. The addition of an arrow-head at Y, and the line itself, show that the direction of the force is horizontally towards the right. The line is drawn a definite length, 3 in., and, by adopting a suitable force-scale such as 1 in. to 1 lb., may represent a force of 3 lb. The same line, by adopting other force-scales, may represent tons, hundredweights, or any other desirable unit. For example, it may represent 3 tons (1 in. to 1 ton), or 6 cwt. (1 in. to 2 cwt.), &c. Thus, if Y is known to be the point of application, and the force-scale is 1 in. to 1 lb., the straight line XY will represent a force of 3 lb. acting on a body and moving it, or tending to move it, horizontally to the right. The force may either be pushing the body at the point X or pulling it at the point Y; it will be understood that a metal

bar or similar stiff material connected to any point in the line of the force would enable the body to be either pulled or pushed, whereas a flexible connection could be used only to pull the body.

Exercises, with answers, on p. 155.

CHAPTER II

FORCE, WORK, AND POWER

FORCE.—As previously explained, force is that which causes, or tends to cause, motion or change of motion in the matter upon which it acts. It is measured in gravitational units, the pound avoirdupois, or any convenient multiple of the pound. The main point to remember at present is that it is measured by a simple unit involving one kind of dimension only, viz. pounds.

WORK.—When any force acts on a body, and causes that body to move, the force is said to have done work. In the measurement of work done there is no question of the time which the force takes to complete the movement. If we assume a loom to be running at 150 picks per minute, the full cycle of operations, so far as the revolution of the crank C, fig. 1, is concerned, is completed in the $\frac{1}{150}$th part of a minute, or in one revolution of the usual driving pulley. If the speed of the loom for any reason is reduced to 140 picks per minute, the same quantity of work is still done in one revolution of the pulley; the sole difference is that the complete cycle of operations has occurred in the $\frac{1}{140}$th part of a minute. The speed or rate at which the operations take place does not,

however, alter in any way the quantity of work done in one revolution of the pulley.

If the force simply moves the body through any distance, great or small, in any period of time, work is done. Two things only are considered: (*a*) the size or magnitude of the force, and (*b*) the distance through which the force acts, or through which it overcomes resistance.

UNIT OF WORK.—Work done is therefore measured by a compound unit, involving force and distance. The unit of force is 1 lb., while the unit of distance is taken to be 1 ft. Combining these two units, the foot and the pound, we obtain the unit of work, known as the foot-pound or ft.-lb. Thus:

$$1 \text{ ft.} \times 1 \text{ lb.} = 1 \text{ ft.-lb.} = 1 \text{ unit of work,}$$
$$10 \text{ ft.} \times 5 \text{ lb.} = 50 \text{ ft.-lb.} = 50 \text{ units of work,}$$

or, in general,

$$x \text{ ft.} \times y \text{ lb.} = xy \text{ ft.-lb.} = xy \text{ units of work.}$$

The foot-pound may therefore be defined as the amount of work done when a force overcomes a resistance of 1 lb. through a distance of 1 ft.

Example 1.—In pushing a loaded yarn-barrow along the level floor of a mill, the operative has to exert a constant force of 50 lb. How much work does he do in 5 min., supposing he travels at the rate of 3 miles per hour?

$$\frac{3 \text{ miles per hour}}{60 \text{ min. per hour}} = \frac{3 \times 5280 \text{ ft.}}{60 \text{ min.}}$$
$$= 264 \text{ ft. per minute.}$$
$$264 \text{ ft. per minute} \times 5 \text{ min.} = 1320 \text{ ft. in } 5 \text{ min.}$$
$$\text{Work done in } 5 \text{ min.} = 50 \text{ lb.} \times 1320 \text{ ft.}$$
$$= 66{,}000 \text{ ft.-lb.}$$

Or, more briefly:

Work done in 5 min.

$$= \frac{3 \text{ miles per hour}}{60 \text{ min. per hour}} \times \frac{5280 \text{ ft.}}{1} \times \frac{5 \text{ min.}}{1} \times \frac{50 \text{ lb.}}{1}$$

$$= \frac{3 \times 5280 \times 5 \times 50}{60}$$

$$= 66{,}000 \text{ ft.-lb.}$$

Example 2.—A pump at work in a bleachfield is employed in raising water from a well to an overhead

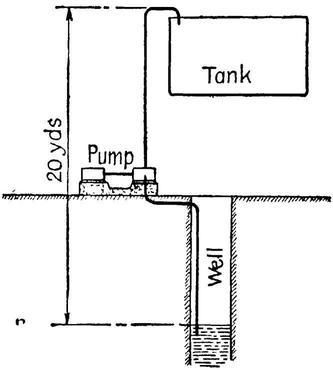

Fig. 5

tank; find the amount of work done in raising 136 c ft. of water to a height of 20 yd. Refer to fig. 5, and assume 1 c. ft. of water $= 62\frac{1}{2}$ lb.

Work done = Weight lifted in pounds × distance moved in feet

$= (136 \text{ c. ft.} \times 62\frac{1}{2} \text{ lb. per cubic foot}) \text{ lb.} \times (20 \text{ yd.} \times 3 \text{ ft. per yard}) \text{ ft.}$

$= (136 \times 62\frac{1}{2} \times 20 \times 3) \text{ ft.-lb.}$

$= 510{,}000 \text{ ft.-lb.}$

Example 3.—A hole is punched in a piece of woven belting $\frac{1}{4}$ in. thick, the pressure exerted being estimated at 50 lb. Assuming the resistance to be uniform, find the number of foot-pounds of work done. (See fig. 6.)

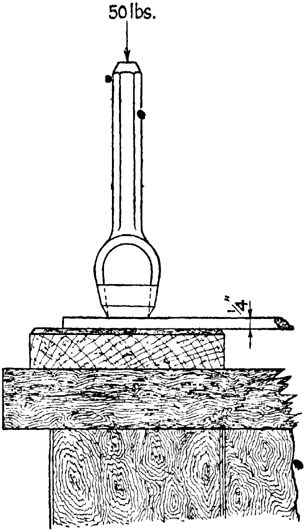

Fig. 6

Work done = Distance in feet × force or resistance in pounds

$= \frac{1}{4}$ in. ÷ 12 in. per foot × 50 lb.

$= (\frac{1}{4} \times \frac{1}{12} \times \frac{50}{1})$ ft.-lb.

$= \frac{50}{48} = 1 \cdot 042$ ft.-lb.

Example 4.—The plunger of a force-pump in a certain dye-house is 8 in. in diameter, the length of its stroke is 2 ft. 6 in., and the pressure of the water is

50 lb. per square inch. Find the number of units of work done per stroke. (See fig. 7.)

Work done = Distance in feet × force in pounds × total pressure on plunger
= 2 ft. 6 in. stroke × 50 lb. per square inch × area of 8-in.-diameter circle
= $2\frac{1}{2}$ ft. × (50 × ·7854 × 8 in. × 8 in.) lb.
= ($2\frac{1}{2}$ × 50 × ·7854 × 8 × 8) ft.-lb.
= 6283·16 ft.-lb.

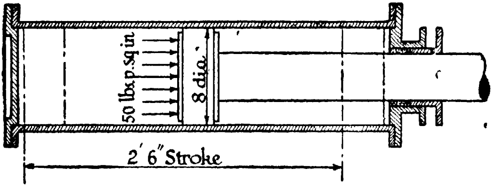

Fig. 7

In every case the student should attempt to make a diagrammatic sketch to illustrate the problem, somewhat similar to those given above in connection with the examples worked out. This practice will ultimately give him a real facility in sketching, and enable him readily to grasp and visualize the particular problem; the importance of being able to sketch rapidly cannot be too strongly emphasized.

Variable Resistance.—In all the examples taken above, and in many other cases which arise in practice, the resistance to be overcome, and consequently the magnitude of the force employed, is a constant quantity. On the other hand, however, in a large number of examples, the resistance to be overcome is a variable one, so that the force required to overcome the resistance is also a variable one. In a certain proportion

of cases the variation is great and difficult to estimate. For instance, the force required to drive a machine such as a loom or a spinning mule varies at different points in the cycle of operations. If a loom is turned round by hand, the operation is comparatively easy until the pick occurs, at which point considerable force is necessary to get the shuttle to move out of the box. Once the shuttle is sent across, the resistance to motion is again comparatively slight until the occurrence of the beat-up, at which point a stiffness is again felt, not so great as that occurring at the pick, but still greater than usual.

In all such cases, before work done can be calculated, the average resistance must be known. This resistance may be found, as a rule, only by experimental means. The results may then be tabulated, and the average found by calculation; squared paper and graphical methods in certain cases may be used with advantage.

In certain other instances, however, although the resistance varies, it does so in a perfectly regular and uniform manner, and the average resistance is then easily found if the first and last values are known. To this section belong such problems as those which follow immediately, and which illustrate the principle involved; their demonstration will aid the student in working out others of a similar nature.

Example 5.—The surface of the water in a dye-house storage-tank is at a depth of 10 ft., and when 500 gall. have been pumped out the surface is lowered to 14 ft. Find the number of units of work done by the pump. Refer to fig. 8; 1 gall. of water = 10 lb.

The water on the surface AB is lifted a distance of 10 ft.
" " CD " " 14 "

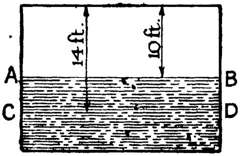

Fig. 8

All the layers of water between these two surfaces are lifted distances varying between 10 ft. and 14 ft. The *average* distance through which the whole of the water is lifted is the average of 10 ft. and 14 ft., so that:

Work done = Weight lifted in pounds × average distance in feet.

$= (500 \text{ gall.} \times 10 \text{ lb. per gallon}) \times \left(\frac{10 + 14}{2}\right) \text{ ft.}$

$= 500 \times 10 \times 12$

$= 60{,}000$ ft.-lb.

Example 6.—A chain, weighing 30 lb. per fathom, is employed to raise a bale of cotton weighing 500 lb. to a height of 36 ft. Find the number of units of work done if the load is raised to within 6 ft. of the point of suspension. (See fig. 9.)

1 fathom = 2 yd. = 6 ft.

Work done in raising bale

= Load in pounds × distance in feet

= 500 lb. × 36 ft.

= 18,000 ft.-lb.

The work done in raising the bale is thus easily found as the resistance, i.e. the weight of the bale, is con stant. Notice, however (see fig. 9), that, as the bale i raised, the length of chain suspended from the pulley and consequently the weight of the chain being raised decreases gradually. The weight of chain bein raised thus varies continuously and uniformly through out the operation, being a maximum at the beginnin and a minimum at the end. At the beginning ther

are 36 ft. + 6 ft. = 42 ft. of hanging chain; and at the end only 6 ft. of hanging chain.

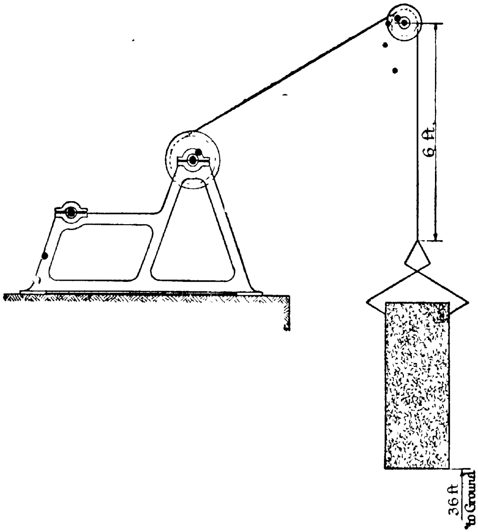

Fig. 9

At beginning:

$$\frac{42 \text{ ft.}}{6 \text{ ft. per fathom}} \times 30 \text{ lb. per fathom} = 210 \text{ lb. of chain.}$$

At end:

$$\frac{6 \text{ ft.}}{6 \text{ ft. per fathom}} \times 30 \text{ lb. per fathom} = 30 \text{ lb. of chain.}$$

Work done in raising the chain

$$= 36 \text{ ft.} \times \left(\frac{210 + 30}{2}\right) \text{ lb.}$$
$$= 36 \text{ ft.} \times 120 \text{ lb.}$$
$$= 4320 \text{ ft.-lb.}$$

Total work done

= Work done on bale + work done on chain
= 18,000 ft.-lb. + 4320 ft.-lb.
= 22,320 ft.-lb.

DIAGRAMS OF WORK.—It has been shown that any force may be represented in magnitude by the length of a straight line, and that lines may be made to represent any length, weight, &c., if a suitable scale is chosen. Force in pounds and distance in feet are the

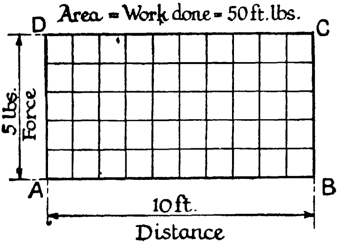

Fig 10

respective components of work done, measured in foot-pounds. If any *area* be taken, such as the rectangle ABCD in fig. 10, the length AB or CD may be chosen to represent any distance to scale, while the width or height AD or BC may likewise be made to represent any force to scale. Now the area of the rectangle is AB × BC, or CD × AD. But if AB represents distance in feet, and BC represents force in pounds, the area of the rectangle ABCD will represent work done in foot-pounds. Such a figure as the rectangle ABCD (fig. 10), is then spoken of as a diagram of work, its area representing to scale the work done in foot-pounds. Suppose, for example, AB represents a distance of 10 ft., and BC represents

a force of 5 lb., the area of ABCD correspondingly represents the work done by a force of 5 lb. in overcoming a resistance through a distance of 10 ft. The work is thus the product of the two—force and distance—and is therefore represented by:

$$10 \text{ ft.} \times 5 \text{ lb.} = 50 \text{ ft.-lb.}$$

This is one of the simplest examples of representing work done by an area, but the principle holds good

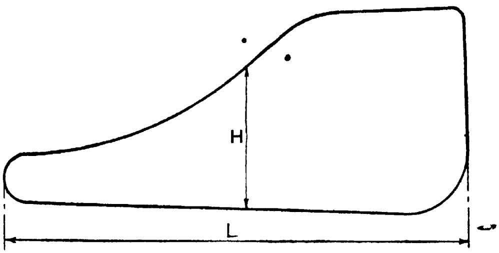

Fig 11

irrespective of the shape of the diagram. For example, steam and other engine diagrams, drawn by a heat-engine indicator, are often very irregular figures; the general form of a steam-engine diagram is shown in fig. 11, where the length L of the diagram represents the stroke of the piston in feet to scale; the height of the diagram taken at any point in its length, such as H, represents, also to scale, the steam pressure acting on the face of the piston at that moment. The area of the irregular diagram represents the total work done by the engine in 1 stroke of the piston. It is convenient to note at this stage that the average pressure throughout the stroke is the average height of the diagram.

Diagrams of work done may be drawn to suit all cases; they are, in general, unnecessary when the result can be obtained by finding the product of two numbers as in connection with fig. 10, but are very useful when the force doing the work is continually varying in an irregular manner or intermittently varying somewhat as in fig. 11. The following examples are representative of the various forms in which these problems may occur in practice.

Example 7.—WORK DONE AGAINST UNIFORM RESISTANCE.—A hoist, or elevator, is used to raise a

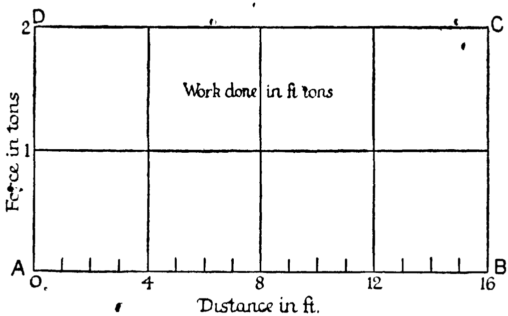

Fig. 12

textile machine weighing 2 tons to the upper floor of a mill, a distance of 16 ft. Show by a diagram of work the amount of work done in raising the load.

Let 1 in. = 1 ton = the force scale,
and 1 in. = 4 ft. = the distance scale.

Draw the line AB, fig. 12, 4 in. long to represent the distance of 4×4 ft. = 16 ft. to the given scale. Set up AD vertically to a height of 2 in. to represent 2 tons to the given scale of 1 in. = 1 ton. Complete

the rectangle ABCD, the area of which now represents the work done in raising the machine to the desired height.

$$\begin{aligned}
\text{Area of rectangle} &= \text{length} \times \text{breadth (or height)} \\
&= 4 \text{ in.} \times 2 \text{ in.} = 8 \text{ sq. in.,} \\
\text{but each square inch} &= 1 \text{ ton} \times 4 \text{ ft.} \\
&= 4 \text{ ft.-tons.} \\
\therefore\ 8 \text{ sq. in.} &= (8 \times 4) \text{ ft.-tons} \\
&= 32 \text{ ft.-tons, the work done, or}
\end{aligned}$$

directly:

$$\begin{aligned}
\text{Work done} &= 16 \text{ ft.} \times 2 \text{ tons} \\
&= 32 \text{ ft.-tons.}
\end{aligned}$$

Example 8.—Work done against a Uniformly Increasing Resistance.—Refer again to Example 6, p. 20, and suppose that the chain used on the bale is coiled home to the point of suspension, and is then lowered until 42 ft. are hanging free; find the work done in the lowering of the chain, and show by a diagram how the force varies.

At the beginning, the effective weight of the chain is 0. At the end, the weight of the chain at 30 lb. per fathom is

$$\frac{42 \text{ ft.} \times 30 \text{ lb.}}{6 \text{ ft. per fathom}} = 210 \text{ lb.}$$

So that the force will vary continuously and regularly from 0 at the beginning to 210 lb. at the end.

$$\text{Scales used:} \begin{cases} \text{Force} = 1 \text{ in. to } 100 \text{ lb.} \\ \text{Distance} = 1 \text{ in. to } 10 \text{ ft.} \end{cases}$$

Draw the line AB, fig. 13, 4·2 in. long to represent the distance of 42 ft. Set up BC vertically a height

of 2·1 *in. to represent the* force at the end of the operation. The force at the beginning is 0, represented by the point A; if AC be joined, the line AC shows how the lowering force continuously and uniformly varies from 0 to 210 lb.

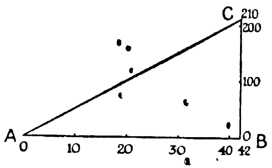

Fig. 13

$$\begin{aligned}\text{Work done} &= \text{area of diagram}\\ &= \text{area of triangle ABC}\\ &= \tfrac{1}{2}\text{ base} \times \text{altitude}\\ &= \frac{42\text{ ft.}}{2} \times 210\text{ lb.}\\ &= 4410\text{ ft.-lb.}\end{aligned}$$

Example 9.—WORK DONE AGAINST A UNIFORMLY DECREASING RESISTANCE.—Take the same case as given in Example 8, but show by means of a diagram the work done in bringing the end of the chain home to the point of suspension through a distance of 42 ft.

At the beginning the force required

$$\begin{aligned}&= \frac{42\text{ ft.}}{6\text{ ft. per fathom}} \times 30\text{ lb. per fathom}\\ &= 210\text{ lb.}\end{aligned}$$

At the end the force required = 0.

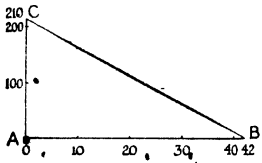

Fig. 14

Between these two extremes the force varies continuously and uniformly. Again, using the same scales as in Example 8, draw AB, fig. 14, 4·2 in. long, to represent 42 ft. Set up AC, 2·1 in.

long, to represent 210 lb., the weight of the chain, and therefore the force required at the beginning. Point B represents the force at the end. Join BC to complete the diagram.

$$\begin{aligned}
\text{Work done} &= \text{Area of diagram} = \text{area of} \triangle \text{ ABC} \\
&= \tfrac{1}{2} \text{ base} \times \text{altitude} \\
&= \frac{42 \text{ ft.} \times 210 \text{ lb.}}{2} \\
&= 4410 \text{ ft.-lb.}
\end{aligned}$$

If the diagrams in figs. 13 and 14 be compared, it will be seen that they are similar, but mirror images of each other, as might be expected from the nature of the problems. Example 8 is concerned with the lowering of the chain, and the present one with the raising of it.

Example 10.—WORK DONE AGAINST A COMBINATION OF UNIFORM AND VARIABLE RESISTANCES.—Refer to Example 6, p. 20, and draw a diagram of the work done in raising the bale of cotton with the chain through a height of 36 ft.

$$\text{Scales used:} \begin{cases} \text{Force} & = 1 \text{ in. to } 200 \text{ lb.} \\ \text{Distance} & = 1 \text{ in. to } 10 \text{ ft.} \end{cases}$$

A commencement may be made by setting up AB, fig. 15, 3·6 in. long, to represent 36 ft. The bale weighs 500 lb., and this weight, and therefore the force required to lift it, is constant. Set out BC horizontally, 2·5 in., to represent 500 lb., and complete the rectangle ABCD, which shows the diagram of work done in raising the bale of cotton only.

At the beginning of the operation the chain hanging free weighs 210 lb., so from AD extended make the line DE 1·05 in. to represent 210 lb. At the end of

the operation the part of the chain hanging free weigh 30 lb., so on BC continued make CF 0·15 in long, to represent 30 lb *Complete the diagram by joining EF. The trapezium* CDEF represents the work done in raising the chain.

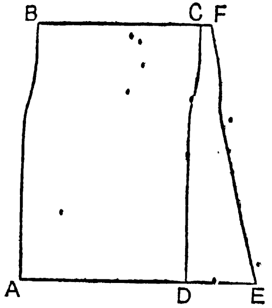

Fig. 15

The complete diagram AEFB, which is also a trapezium, represents diagrammatically the whole work done in the operation, and its area is therefore obtained by the usual general rule.

$$\begin{aligned}
\text{Area} &= \tfrac{1}{2}\text{ sum of parallel sides} \times \text{perpendicular distance between them} \\
&= \frac{(500 + 210)\text{ lb.} + (500 + 30)\text{ lb.}}{2} \times 36\text{ ft.} \\
&= \frac{710\text{ lb.} + 530\text{ lb.}}{2} \times 36\text{ ft.} \\
&= 620\text{ lb.} \times 36\text{ ft.} \\
&= 22{,}320\text{ ft.-lb.}
\end{aligned}$$

In the diagram in fig. 15 force is represented horizontally, while height or distance is represented vertically; this is technically known as a "force base", whereas the previous diagrams are drawn on a "distance base". It is immaterial which is used as a base, and it is usual to choose that which appears to be the more convenient. It may be further remarked that the complete diagram is really a combination of two, the first being a simple rectangular diagram, such as is given in Example 7, and the

other a variation of that used in Example 9, dealing with a uniformly decreasing resistance.

Example 11.—WORK DONE AGAINST AN IRREGULARLY VARYING RESISTANCE.—*A steam-engine diagram is represented in fig. 16. If the stroke of the piston is 4 ft., the scale of the diagram* $\frac{1}{40}$, and the area of the face of the piston 400 sq. in., find the work done per stroke.

The scale marked $\frac{1}{40}$ is the pressure scale, and implies that $\frac{1}{40}$ of an inch in height corresponds to 1 lb.

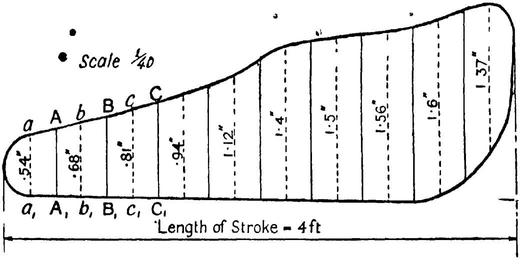

Fig. 16

of steam pressure in the engine cylinder on each square inch of the piston area, in other words, 1 in. of height = 40 lb. per square inch of steam pressure. The work done is given by the area of the diagram, and, as previously mentioned (p. 23), the area of the diagram may be found by multiplying its length by the average height. Any irregular figure may be treated in this way (see Woodhouse & Brand's *Textile Mathematics*, Part II, p. 12).

The area of the diagram may be found directly by means of a planimeter, but, as the mean or average pressure is required for calculating the horse-power of an engine (see later), it is more convenient for present

purposes to obtain the average height of the diagram which corresponds to the mean or average pressure throughout the stroke of the piston. This average height may be found in a number of ways; a simple and convenient method is that shown in fig. 16. The length of the diagram is divided into ten equal parts denoted by the lines AA_1, BB_1, &c., and the average height of each of the sections of the diagram is measured along the dotted vertical lines aa_1, bb_1, &c. These ten measurements, which appear close to the lines in the diagram, should be collected to find the average; thus,

Average height

$$= \frac{\cdot54 + \cdot68 + \cdot81 + \cdot94 + 1\cdot12 + 1\cdot4 + 1\cdot5 + 1\cdot56 + 1\cdot6 + 1\cdot37}{10}$$

$$= \frac{11\cdot52}{10} = 1\cdot152 \text{ in.}$$

But each inch $= 40$ lb. per square inch.

$\therefore$ $1\cdot152$ in. $= (40 \times 1\cdot152)$ lb. per square inch

$= 46\cdot08$ lb. per square inch pressure.

The total force acting on the piston is thus

$46\cdot08$ lb. per square inch $\times$ 400 sq. in. $=$ 18,432 lb. per stroke.

An *average* force of 18,432 lb. thus overcomes a resistance through a distance of 4 ft. Now,

Work done $=$ Resistance or force in pounds $\times$ space or distance in feet.

$= R \times S$

$= 18{,}432 \text{ lb.} \times 4 \text{ ft.}$

$= 73{,}728$ ft.-lb. per stroke.

Squared paper, preferably ruled in inches and tenths of an inch, will be found most useful for the speedy

and accurate drawing of diagrams of work, such as those in the above examples.

POWER.—Power may be defined as the *rate* at which work is done. It has already been stated that force involves a simple unit only, viz. pounds. On the other hand, *the unit of work is not simple but compound, since it involves the* product of force and distance, work being done when a resistance is overcome through a certain distance. When measuring the rate at which a particular force overcomes a resistance, time must be taken into account, and in doing so there is involved the more complex unit of power. If it be known that a force of, say, 10 lb. overcomes a resistance through a distance of, say, 50 ft. in a certain definite space of time, such as 1 min., the rate at which the work is being done is known, and this rate is termed the *power* of the force. If the work were done in ½ min. instead of 1 min., the force would be twice as powerful as in the longer period, since it is doing the same work in half the time, i.e. it is working at twice the rate, or twice as fast.

UNIT OF POWER.—The unit of power in English-speaking countries is called the horse-power, commonly abbreviated to H.P., and often to HP. It was so called by James Watt, whose steam-engines were often used to do work previously done by horses. He arbitrarily fixed the unit as being equal to 33,000 ft.-lb. of work per minute, or 550 ft.-lb. per second.

One horse-power may therefore be defined as the activity or rate of action of a force which does work at the rate of 33,000 ft.-lb. per minute.

A force of	100 lb.	acting through	330 ft.	in	1 min.	= 1 H.P.
,,	330	,,	50	,,	½ ,,	= 1 H.P.
,,	550	,,	100	,,	100 sec.	= 1 H.P.

A general expression, which is easily remembered, is

$$\text{H.P.} = \frac{RS}{33{,}000},$$

where R is the resistance overcome in pounds, and S is the space passed through in feet *per minute*.

Example 12.—Referring to Example 1, p. 15, it is seen that the operative does 66,000 ft.-lb. of work in 5 min., or, to put it another way, he exerts a force of 50 lb. through a distance of 1320 ft. in 5 min. At what rate is he doing work, i.e. what is the equivalent horse-power?

$$\text{H.P.} = \frac{RS}{33{,}000} = \frac{50 \text{ lb.} \times 1320 \text{ ft.}}{33{,}000 \times 5 \text{ min.}}$$

$$= \frac{50 \times 1320}{33{,}000 \times 5} = \frac{2}{5} = 0{\cdot}4 \text{ H.P.}$$

Example 13.—With reference to Example 2, p. 16, suppose the bleachfield pump raises 136 c. ft. of water a height of 20 yd. in 4 min. What horse-power is being exerted in raising the water?

$$\text{H.P.} = \frac{RS}{33{,}000} = \frac{(136 \times 62\tfrac{1}{2}) \text{ lb.} \times (20 \times 3) \text{ ft.}}{33{,}000 \times 4 \text{ min.}}$$

$$= \frac{136 \times 62{\cdot}5 \times 20 \times 3}{33{,}000 \times 4} = 3{\cdot}86 \text{ H.P.}$$

This, of course, is the power expended in the actual raising of the water. Friction in the pipes, friction in the working parts of the pump, imperfect working of the valves, &c., would all contribute to increase considerably the actual horse-power required.

Example 14.—In Example 10, p. 27, it was shown that the work done in raising the bale of cotton and the chain is altogether 22,320 ft.-lb. If the operation is completed in $\frac{1}{3}$ min., what horse-power is expended?

$$\text{H.P.} = \frac{\text{RS}}{33{,}000} = \frac{(500 + \frac{210 + 30}{2})\ \text{lb.} \times 36\ \text{ft.}}{33{,}000 \times \frac{1}{3}\ \text{min.}}$$

$$= \frac{620\ \text{lb.} \times 36\ \text{ft.}}{33{,}000 \times \frac{1}{3}\ \text{min.}} = \frac{620 \times 36 \times 3}{33{,}000 \times 1} = 2{\cdot}03\ \text{H.P.}$$

Here, again, the resulting figure is merely the horse-power expended in the actual raising of the bale and chain, no account being taken of friction, &c.

Example 15.—If the indicator diagram in fig. 11 is the average of a large number of working strokes, find the horse-power which the engine develops when running at 200 strokes per minute (100 rev. per minute).

$$\text{H.P.} = \frac{\text{RS}}{33{,}000}$$

$$= \frac{(46{\cdot}08\ \text{lb. per sq. in.} \times 400\ \text{sq. in.})\ \text{lb.} \times (4\ \text{ft.} \times 200\ \text{strokes})\ \text{ft.}}{33{,}000}$$

$$= \frac{46{\cdot}08 \times 400 \times 4 \times 200}{33{,}000} = 446{\cdot}84\ \text{H.P.}$$

In the case of any type of heat-engine, a special form of the general equation is more often used; it may be obtained as under:

Let P = the mean effective pressure in pounds per square inch.
A = the effective area of the piston in square inches.
L = the length of stroke in feet.
N = number of working strokes per minute.

Now, $$\text{H.P.} = \frac{\text{RS}}{33{,}000},$$

and, in the case of the engine,

$$\text{R} = (\text{P} \times \text{A}) \text{ and } \text{S} = (\text{L} \times \text{N}), \text{ so that}$$

$$\text{H.P.} = \frac{\text{PA} \times \text{LN}}{33{,}000} = \frac{\text{PALN}}{33{,}000}.$$

With a view to enable the student to remember the numerator, it is usual to change the positions of A and L, so that the equation may read:

$$\text{H.P.} = \frac{\text{PLAN}}{33{,}000}.$$

Exercises, with answers, on p. 155.

CHAPTER III

STATICAL DEFINITIONS

It has already been shown that the visible effect of force is motion, and that, where a force is applied and no motion occurs, there is, nevertheless, a tendency to produce motion. The gable or framing of a carding engine or a loom, for instance, does not move when the remainder of the machine is in motion; the gable is, however, subjected to many forms of force-action, and, to fulfil its function, must be made strong enough to resist these force-actions. In other words, the forces acting on the gable tend to move it, and are prevented from doing so by the design and fixing of the gable, and the force-actions are resisted by stresses in the material. In such machines, and in several others, there are many different forces, and if the machine were not fixed to the floor, those which preponderate in a certain direction would cause the whole machine to move gradually in the corresponding direction. Forces such as stresses in materials, and which do not actually cause motion, are dealt with by that branch of science known as Statics, and it will prove convenient to take up at this stage several useful

and general definitions, which will render subsequent chapters more easily understood.

FORCES IN EQUILIBRIUM.—1. When any number of forces acting on a body neutralize the effect of one another, the forces are said to be in equilibrium. Another way of stating the same proposition is: If any number of forces be applied to a body and the body remains in the same condition as to rest or motion as it was previous to the forces being applied, then the forces must be balanced, i.e. they must be in equilibrium.

An ordinary balance or scale, such as is used in retail shops, may be used as a simple illustration of this statement. The scales, if placed on a horizontal surface and perfectly adjusted, will remain balanced and motionless. If a 1-lb. weight is placed in one pan, the scale will immediately move, since all the forces acting on it are not in equilibrium. If a second 1-lb. weight is placed in the other pan, the scale will again move, in the opposite direction, and, probably after a few oscillations, will ultimately come to rest in the same position as obtained before the first weight was applied. All the forces acting on the scale will again be in equilibrium, i.e. in a state of balance.

2. Two or more forces which are in equilibrium may be applied to a body, or removed from a body, without altering its condition either of rest or of motion.

The scale illustration may again be used. It is obvious that if the two 1-lb. weights—which are two forces in equilibrium—be removed simultaneously, the scale will still remain in the same condition, as to rest, as obtained before the removal of the weights. Or if 3, 4, 5 .. n different pound-weights be placed in each pan, the same state of equilibrium will result, because

the two sets of forces or weights are absolutely equal, and balance each other.

3. Forces which are equal in magnitude but opposite in direction will destroy the effect of each other. (This is actually the case with the two sets of weights, although the reason for the effect of their oppositely-directed action has not yet been considered.) The converse is also true, i.e. no system of forces can destroy the effects of another system of forces, unless they are equal in magnitude and opposite in direction.

A slight study of the scale illustration used above will render these statements self-evident.

4. A force will produce the same effect at *any* point in its line of action. If any particular force is known to produce the same effects at two different points, the straight line joining these two points will give the line of action or the direction of the force.

This fourth definition is obvious, and needs no further explanation.

Action and Reaction.—When any fixed rigid body is acted on by a force, there is immediately produced *in* that body a secondary force equal in magnitude but opposite in direction to the primary force. This secondary force is termed a force of reaction, the primary one being regarded as a force of action; the chief point to grasp at present is that one cannot exist without the other.

Let the student take a weight of any kind and place it on a horizontal surface such as a table. The weight will remain on the table; it does not move about the table, neither does it fall through it or rise up from it. The reason is that whenever the weight is so placed on the table, it immediately exerts a vertically downward force on the table, and *tends* to fall through it; it is prevented from doing so by a secondary force or

reaction being immediately set up in the table which resists the primary force. If the weight and the table do not move, this secondary force must be equal in magnitude but opposite in direction to the primary force. Again, a light celluloid or other ball can be kept from falling by a suitable jet of water. If the jet of water were constant in force and its particles suitably distributed, the ball would remain suspended. It rises and falls intermittently, because the force of the water jet is variable. Action and reaction are again equal but oppositely directed. Hence, as it is often stated: Action and Reaction are equal in magnitude, opposite in direction, and neutralize each other's effects.

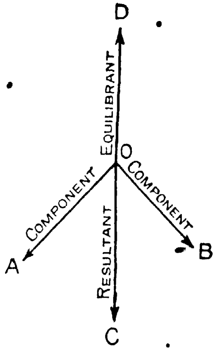

Fig 17

RESULTANTS AND COMPONENTS. — If a number of forces acting on any body can be replaced by a *single* force with the same result, this single force is termed the RESULTANT of the individual forces, and the individual forces are called COMPONENTS. The operation of finding the resultant of a system of forces is termed *Composition of Forces*, while the operation of finding components is termed *Resolution of Forces*. As an illustration, suppose that two forces OA and OB, fig. 17, are acting obliquely downwards from a point O in a body, these two forces may be replaced by a single force denoted by the line OC. The exact direction and magnitude of this single force OC will naturally be influenced by the exact magnitudes and directions of the pair of forces OA and OB. For

instance, if OB is a greater force than OA, then OC will be nearer to OB than to OA. The forces OA and OB are components of the force OC; the force OC is the resultant of the two forces OA and OB.

EQUILIBRANT.—If a number of forces act on a body and move or tend to move it, the *single* force which will neutralize this motion or tendency to motion is termed the *Equilibrant*, or balancing force.

With reference to fig. 17, it is clear that the downward pull of the two forces OA and OB could be neutralized by the upward pull of a suitably dimensioned force, such as that indicated by OD. The exact magnitude and direction of this upward force will again be influenced as a matter of course by the magnitudes and directions of the component forces. The force OD is thus the equilibrant of the two forces OA and OB. In any particular system of forces, it is evident from the given definitions, that the Resultant and the Equilibrant must be equal in magnitude but opposite in direction, as indicated in fig. 17.

Exercises, with answers, on p. 157.

CHAPTER IV

CENTRES OF GRAVITY

MATTER.—When matter is examined from the point of view of structure, this structure is found to be somewhat complex. A piece of wrought iron, for instance —wrought iron being a practically pure substance— may be subdivided by abrasive action of any sort into extremely small pieces termed particles, literally "little parts". These particles can be further subdivided into extremely minute pieces termed molecules, literally

"little masses"; a molecule is defined by chemistry as the smallest weight of matter that can retain the original properties of the matter; these molecules are still further subdivided into atoms, which are defined by chemists as the smallest particles of matter that can take part in a chemical change. At one time it was thought that atoms were indivisible particles, but recent study in this connection has led to the belief that atoms themselves have a more or less complex structure, being composed of ions; the theory itself is referred to as the Ionic Theory.

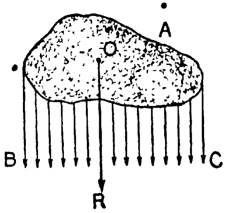

Fig. 18

FORCE OF GRAVITY.—Gravity is the name given to that force of attraction which all bodies possess for one another. This force varies with many factors, the chief of which are the distance between the bodies and their relative sizes and densities. The sun, moon, earth, stars, and other constituents of the solar system are all maintained in position in space by the forces of attraction exerted by one and all of the bodies. This property of attraction is briefly termed the force of gravity.

WEIGHT.—Any bodies near the earth are attracted by the force of gravity towards the centre of the earth. This gravitational attraction causes a body to possess that which is termed weight. The weight of a body is thus a measure of the gravitational attraction of the earth upon the body.

CENTRE OF GRAVITY.—Any body is made up of a number of particles, and the force of gravity acts upon each particle. This is represented diagrammatically in fig. 18, where the arrow-headed lines BC represent

the downward pull of the force of gravity upon the particles composing the body A. The weight of the body may be considered as the *resultant* of all these downward parallel forces.

If the weight of the body could be concentrated at one point, this resultant would pass through the point; this assumption is also diagrammatically shown in fig. 18, where R is the resultant, and O is the particular point. This point is termed the CENTRE OF GRAVITY (often abbreviated to C.G.).

In many kinds of calculations in mechanics the position of the centre of gravity is all important, since it may be assumed that the whole weight of any regular or irregular body can be concentrated and act at its centre of gravity.

The exact position of the centre of gravity in any body is most correctly found by mathematical means, but a fairly advanced knowledge of mathematics is required except for the more simple geometrical solids. The purpose of an elementary work will be served by demonstrating how the centres of gravity may be found in the case of simple objects by experimental means. More correctly speaking, the following examples show how to find the *centres of areas* of plane surfaces of various shapes, but from the results obtained useful general deductions may be made.

The student should obtain six pieces of smooth regular cardboard, and from them cut: (1) a rectangle; (2) a square; (3) a parallelogram; (4) a triangle; (5) a semi-circle; and (6) any irregularly-shaped surface with straight and curved edges. He should further provide himself with a small leaden plummet or brass plumb-bob and line, and two ordinary sewing-needles, one small and one large.

Example 16.—Determine experimentally the centre of area of a rectangle.

METHOD.—The method described is perfectly general, and should be followed in the present and the five following examples:—

1. Pierce a small needle through the rectangle at *any* point near one edge, and suspend it from a door, wooden table, board, or the like, so that the cardboard may swing on the needle quite freely.

2. Hang a plumb-line on the needle, so that its cord lies as close as possible to the cardboard without actually touching it.

3. Set the card swinging, and allow it to come to rest. The centre of area must now be directly under the point of suspension. Mark the line of the plumb-bob near the edge opposite the point of suspension; remove the card, and draw a straight line between the two points. The centre of gravity is somewhere on this line.

4. Repeat the third operation, using a new point of suspension, and obtain a second line which intersects the first at a point. This point of intersection is the required centre of area.

5. Check the result by using a third point of suspension. The line thus obtained should pass through the same point. As a further check, lay the card on any flat surface, push the large needle into the centre of gravity obtained, and lift the card slowly upwards. If the point has been correctly found, the card will lift horizontally and have no tendency to tilt downwards at any part of its edge.

With the aid of a set-square, draw the two diagonals of the rectangle. The intersection of the diagonals will coincide with the point found.

Make a sketch of the apparatus, similar to fig. 19,

and note that the centre of area of a rectangle is at the intersection of its diagonals.

Example 17.—Follow out the instructions given in Example 16, using the square cardboard, and verify the truth of the following statement:—The centre of area of a square is at the intersection of its diagonals.

Note that this is simply a variation of Example 16, since a square is a special form of rectangle.

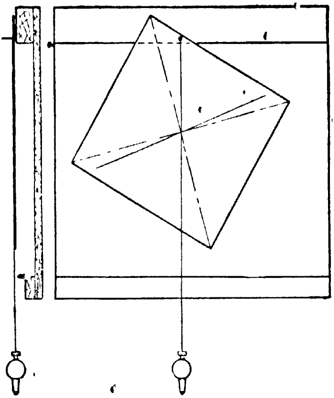

Fig. 19

Example 18.—Again use the instructions in Example 16, using the parallelogram cardboard, and see that the centre of area again coincides with the intersection of the diagonals. This result is to be expected, because a square and a rectangle may be regarded as special forms of parallelograms.

Example 19.—Repeat Example 16 with the triangle. When the centre of area has been found by experimental means, bisect any two sides, and join the middle points of these sides to the opposite angles or points. The intersection of these lines should coincide with the centre of area found.

Thus the centre of area of a triangle is at the intersection of the straight lines drawn from two of the angles to the middle points of the sides opposite these angles.

Example 20.—With the same instructions from Example 16, and with the semicircle, find the centre of area; when this has been established experimentally, draw the radius perpendicular to the base or diameter of the semicircle. The centre of area will be found to lie on this line. This is shown in fig. 20, where AB is the radius, CD the diameter, and G the centre of area.

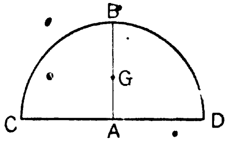

Fig. 20

Now measure AB and AG very carefully, and find the ratio between them. If the experiment has been carefully and correctly conducted, it will be found that:

$$\text{the ratio } \frac{AG}{AB} = \frac{\cdot 4244}{1} = \cdot 4244.$$

The following is the result of an actual experiment:—

AG measured 5·05 cm.
AB ,, 11·90 cm.

$$\frac{AG}{AB} = \frac{5\cdot 05}{11\cdot 90} = \cdot 42437.$$

Mathematically, the correct value is found to be

$$\frac{AG}{AB} = \frac{4}{3\pi}.$$

But, since AB = the radius = R, it may be shown that

$$AG = \frac{4R}{3\pi} \doteq \cdot 4244R.$$

Example 21.—With a final use of the instructions in Example 16, and the irregular figure, find the centre of area by experimental means; the only efficient means of checking, apart from mathematical

calculation, is to suspend the figure from three or four different points, and to see that in every case the plumb-line covers the centre of area originally found.

In the great majority of cases practical men have to deal with solids, and not with surfaces; nevertheless the above results are exceedingly useful. Generally speaking, if a body is symmetrical about any centre line, the centre of gravity lies in that line. If, by any means, two such lines can be found in a body, the centre of gravity is at their intersection. In all bodies or surfaces of regular shape, this condition is easily fulfilled; in such cases, the centre of gravity or of area (or centroid, as it is also termed) coincides with the geometrical centre of the solid or surface.

Exercises, with answers, on p. 157.

CHAPTER V

THE PARALLELOGRAM OF FORCES

In a number of problems in mechanics, it becomes necessary to find the *single* force which will either (1) balance two other forces acting at a point, i.e. an equilibrant; or (2) replace and produce the same effect as two other forces acting at a point, i.e. a resultant.

The method of solution depends upon the truth of what is termed the Proposition of the Parallelogram of Forces; an experimental method of deducing this proposition is given in the following paragraphs.

Example 22.—To deduce from the results of experiment the Proposition of the Parallelogram of Forces.

The apparatus required is simple. It consists of an upright wooden board A, fig. 21, to which are clamped two light, freely-running grooved pulleys B and C. These pulleys should be as frictionless as possible, and are preferably made of aluminium and constructed to run on steel centres. Cords D and E, to the ends of which weights of 20 oz. and 30 oz. are

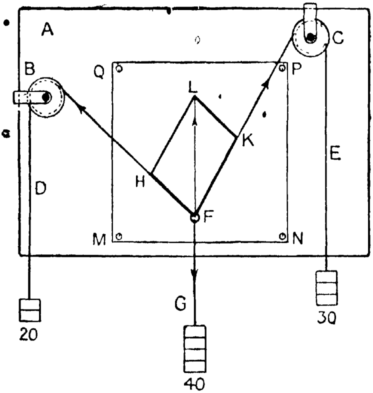

Fig. 21

attached, are passed over the pulleys B and C, and fixed to a small brass-wire ring at F. A third cord G is fitted to the ring as shown, and a weight (40 oz.) hung on to its lower end. Any weights, other than those mentioned, may be used, provided that the cords occupy a position on the board suitable for conducting the experiment satisfactorily. Lastly, a sheet of paper, MNPQ, is pinned behind the cords as indicated. When the apparatus is thus arranged, it is found that the weights, and consequently the cords, tend to occupy definite positions. By raising

the 40 oz. weight slightly, all the parts will move, but, when the hand is withdrawn, they will gradually return to their original positions of rest. The positions of the essential parts when the whole system comes to rest should be carefully noted, and the operation repeated two or three times to see that the positions of rest are always the same.

From the above experiment it may be inferred that the three forces acting in the cords are in equilibrium, the 40 oz. force being regarded as the equilibrant of the two forces of 30 oz. and 20 oz.

Now mark the point at which all three forces meet; this will be the centre of the brass ring F. From this point draw lines along the oblique parts of the cords D and E to show the *directions* of the forces due to the weights of 20 oz. and 30 oz. Choose a suitable force-scale, and mark off lengths to represent their magnitudes; these are indicated by the lengths FH and FK, and they are equal respectively to 20 and 30 units. From point F, set up a vertical line, and on it mark off a length of 40 units to represent 40 oz.; this is indicated by the line FL.

FL evidently represents the *resultant*, since it is equal in magnitude and opposite in direction to the 40 oz. force. Or, in other words, the 20 oz. and 30 oz. forces are components of the 40 oz. force.

Join the points HL and KL, and remove the sheet of paper MNPQ from the board. With the aid of a pair of set-squares see that HL is parallel to FK, and that KL is parallel to FH, i.e. that FHLK is a parallelogram. If the experiment has been accurately conducted, it will be found to be exactly as stated. The figure FHLK is therefore a parallelogram in which two adjacent sides FH and FK represent, in direction and magnitude, the forces of 20 oz. and

30 oz. respectively, meeting at the point F. The diagonal FL of this parallel, that which passes through the same point as the other two forces, represents the resultant of these two forces in direction and magnitude. From this result the following important inference may be drawn.

PROPOSITION OF THE PARALLELOGRAM OF FORCES.—If any two forces, acting in one plane at the same time, be represented in direction and in magnitude by two adjacent sides of a parallelogram, then the resultant of these two forces will be represented in direction and in magnitude by the diagonal of the parallelogram which originates in the point at which the forces meet.

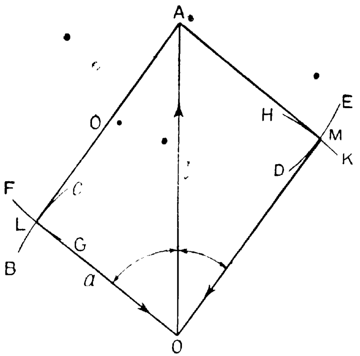

Fig. 22

Example 23.—A vertically upward force of 50 lb. is balanced by two forces of 30 lb. and 40 lb. respectively. Find the directions of the components and the angle between them.

Draw OA, fig. 22, vertically upwards from the point O, and make it $2\frac{1}{2}$ in. long, to represent 50 lb., so that the force-scale is one of 1 in. to 20 lb. With O and A as centres in turn, and with a radius of $1\frac{1}{2}$ in. (30 lb.), draw the arcs BC and DE. With A and O as centres, and a radius of 2 in. (40 lb.), draw the arcs FG and HK, cutting the first two arcs in

points L and M. Join OL, LA, AM, and MO, and form the parallelogram OLAM.

OA (50 lb.) is now the diagonal of a parallelogram, two sides of which, OL and OM, represent 30 lb. and 40 lb. respectively. If OA is the equilibrant of the system and acts upwards, the two components, LO and MO, must act downwards and pass through the point O as indicated by the arrow-heads. If OA had been a resultant, instead of an equilibrant, then OL and OM would have acted upwards.

Diverse means may be employed to find the angle between the components. Thus:

1. Measure $\widehat{LOM}$ with a protractor. If the diagram is large and accurately drawn, $\widehat{LOM}$ will be found to be 90°.

2. Use trigonometrical means. Thus OLA is a triangle, the three sides representing to scale 30, 40, and 50 lb. Now, with respect to $\widehat{LOA}$:

$$\sin \frac{\widehat{LOA}}{2} = \sqrt{\frac{(s-l)(s-a)}{la}},$$

$$\text{and } s = \frac{o+l+a}{2} = \frac{40+50+30}{2} = 60,$$

$$\text{whence, } \sin \frac{\widehat{LOA}}{2} = \sqrt{\frac{(60-50)(60-30)}{50 \times 30}}$$

$$= \sqrt{\frac{10 \times 30}{50 \times 30}} = \sqrt{\frac{1}{5}}$$

$$= \sqrt{\cdot 2} = \cdot 4472.$$

$$\therefore \frac{\widehat{LOA}}{2} = 26^\circ\ 34',$$

$$\text{and } \widehat{LOA} = 53^\circ\ 8'.$$

In a similar manner $\widehat{AOM}$ may be shown to be

36° 52′, so that the whole angle required, or $\widehat{LOM}$ = 53° 8′ + 36° 52′ = 90° = 1 right angle, i.e. the angle between the components OL and OM is 90°.

3. As a check on the above, if $\widehat{LOM}$ is a right angle, $\widehat{LAM}$ is also a right angle, so that $\widehat{OLA}$ and $\widehat{OMA}$ are also each a right angle, i.e. OLAM is not merely a parallelogram, but a rectangle. $\triangle$ OLA is therefore a right-angled triangle, of which OA is the hypotenuse. Consequently

$$\begin{aligned} (OA)^2 &= (OL)^2 + (AL)^2, \\ \text{i.e. } 50^2 &= 30^2 + 40^2, \\ 2500 &= 900 + 1600, \\ 2500 &= 2500. \end{aligned}$$

Whence it is proved that $\widehat{OLA}$ is a right angle, and the remainder, given above, therefore follows.

4. The student may have noticed that in each of the two triangles, OLA and OMA, the constituent sides are proportionately equal to 3, 4, and 5, and hence they are right-angled triangles. (See Woodhouse and Brand's *Textile Mathematics*, Part I, p. 26.)

Example 24.—A bale of cotton weighing 500 lb. is raised by a rope passing over a pulley as indicated in fig. 23. Find the stress on the stud on which the pulley revolves, due to this load.

The two forces are: (1) a vertically downward force of 500 lb. due to the weight of the bale, and (2) a horizontal force (to the right) due to the pull exerted in raising the bale. Friction in the pulley, &c., is neglected. The lines of force-action in the two parts of the rope will, if produced, meet at the point O. The stress or pressure on the stud must be the *resultant* of all the force-actions brought into play.

From any point P, fig. 23, draw PQ vertically downward, parallel to OA and 2 in. long, representing 500 lb. to a scale of 1 in. to 250 lb. Similarly, from point P, draw PR horizontally towards the right, and make it 2 in. long, for 500 lb., to the same force-scale, since the stress in the rope is the same at all parts. Complete the parallelogram PQSR, and draw the diagonal PS, which represents the resultant of PQ and PR in magnitude and in direction. Measure

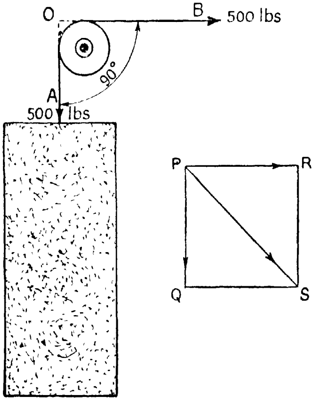

Fig. 23

PS carefully, and find the number of pounds it represents.

$$PS = 2{\cdot}83 \text{ in.} = 707{\cdot}5 \text{ lb.}$$

707·5 lb. is thus the pressure on the stud of the pulley, and it evidently acts obliquely downwards towards the right at an angle of 45°.

As a proof to these figures, PQ and PR are equal in magnitude, while the former is vertical and the

latter horizontal, i.e. QPR is a right angle, whence PQSR is a square. It follows that

$$\begin{aligned} PS &= \sqrt{(PQ)^2 + (PR)^2} \\ &= \sqrt{500^2 + 500^2} \\ &= \sqrt{250{,}000 + 250{,}000} \\ &= \sqrt{500{,}000} \\ &= 707{\cdot}1 \text{ lb.,} \end{aligned}$$

$$\begin{aligned} \text{or } PS &= \sqrt{4 + 4} = \sqrt{8} \\ &= 2{\cdot}828, \end{aligned}$$

$$\text{and } 2{\cdot}828 \times 250 = 707 \text{ lb.}$$

$$\text{Again } \frac{PQ}{PS} = \cos 45.$$

$$\therefore PS = \frac{PQ}{\cos 45} = \frac{500}{{\cdot}7071} = 707{\cdot}1 \text{ lb.}$$

Example 25.—The conditions are identical with those in Example 24, except that the direction of the

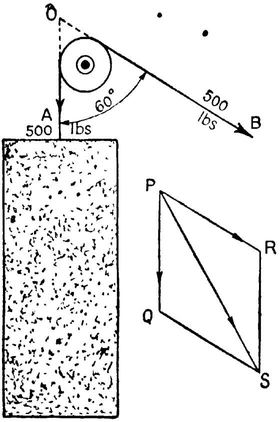

Fig. 24

two parts of the rope form an angle of 60° in place of 90°.

Proceed as indicated in fig. 24, using the same force-scale of 1 in. to 250 lb., and ultimately obtain the resultant PS as in fig. 24. Measure PS carefully, and calculate the force it represents.

$$PS = 3{\cdot}46 \text{ in.} = 865 \text{ lb.;}$$

or,

$$\frac{PQ}{\frac{1}{2}PS} = \sec 30°, \text{ i.e. } \frac{\frac{1}{2}PS}{PQ} = \cos 30°.$$

$$\begin{aligned} PS &= 2 \times PQ \times \cos 30° \\ &= 2 \times 500 \times \cdot 866 \\ &= 866 \text{ lb.} \end{aligned}$$

It will be seen that, as the angle AOB, figs. 23 and 24, gets smaller, the stress on the pin becomes proportionately greater. It may also be inferred that, as the angle increases from 90° to 180°, the stress will become correspondingly less. Indeed, at 180° (two right angles) both parts of the rope would be in one straight line, and the stress on the pin would vanish. Again, when OB becomes parallel to OA, the stress on the pin will be a maximum and equal to 1000 lb.

PARALLELOGRAM OF VELOCITIES.—When any body is in motion, and it is desired to state the *rate* at which the motion is taking place, some such expression as 22 ft. per second, or 15 miles per hour, is used. 22 ft. per second, or 15 miles per hour, is the rate of motion or speed of the body; occasionally the term velocity is used. In mechanics, however, the term velocity means more than a mere speed; it means speed or rate of motion *in a given direction*, so that the velocity of a moving body may be changed:

1. By altering its speed only;
2. By altering its direction only; and
3. By altering both speed and direction.

It is possible for a body to possess at one time more than one velocity. Take, for example, the case of a loom shuttle in its journey from box to box.

1. The shuttle moves, say from left to right, in what is apparently, at first sight, an approximately horizontal direction at a certain speed.

2. As the shuttle crosses the shed the lay moves backwards, carrying the shuttle with it, so that the shuttle moves laterally at a certain speed.

3. The position of the crank and connecting arm in most looms is such that as the lay moves backwards its surface moves downwards, so that the shuttle must move downwards with it.

The actual velocity of the shuttle is thus compounded of at least three velocities while the lay moves backwards only. It may have three different velocities as the lay moves forwards. Each of these three velocities is a *component* of the actual or *resultant* velocity.

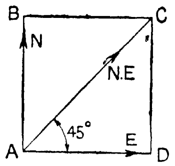

Fig 25

Example 26.—Draw a straight line AC, fig. 25, at 45° to the horizontal. Make this line 2 in. long; it may now be taken to represent the velocity of a ship sailing north-east at a speed of 20 knots (1 knot = 6080 ft. per hour), the velocity scale being 1 in. to 10 knots. If the ship is sailing north-east, its direction is partly north and partly east. Its absolute velocity may therefore be regarded as the resultant of two velocities, the direction of one of which is north, and that of the other east. Now draw AB vertically upwards (north), and AD horizontally to the right (east). Make BC and CD parallel to AD and AB respectively. In this way the resultant velocity AC has been resolved into two components, one vertical or north, and one horizontal or east, as AB and AD respectively.

AB = 1·41 in. = 14·1 knots = northerly velocity.
AD = 1·41 in. = 14·1 knots = easterly velocity.

Stated in another way, the north-easterly velocity of

20 knots is equivalent to two simultaneous velocities, one northerly and one easterly, and each equal to 14·1 knots.

Notice that AC is the diagonal of a parallelogram ABCD, two adjacent sides of which, AB and AD, represent simultaneous velocities of the ship. On these premises, one may found a second proposition, analogous to that of the Parallelogram of Forces.

PROPOSITION OF THE PARALLELOGRAM OF VELOCITIES.—If any two velocities, equal or unequal, occur in a body at one time, and can be represented in direction and magnitude by the adjacent sides of a parallelogram, then the resultant of these two velocities can be represented in direction and magnitude by the diagonal of the parallelogram which originates in the common point of the two adjacent sides.

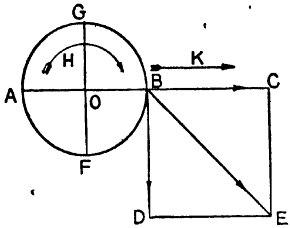

Fig. 26

Example 27.—The carriage of a spinning mule travels 60 in. in 10 sec., i.e. 6 in. per second. By means of a diagram, determine the velocity of a point on the rim of one of the wheels at the instant the point is passing the forward end of the horizontal diameter of the wheel.

With centre O, fig. 26, draw a circle AGBF to represent the wheel; through O, draw AOB, the horizontal diameter. If the wheel be rotating in the direction shown by the arrow H, then B is the required point.

Notice that the wheel has two motions, a revolving motion round point O in the direction indicated by the

arrow H, and a forward straight-line motion along the rails in the direction of the arrow K. The velocity of the point B is thus a resultant of two velocities.

If the speed of the carriage be 6 in. per second, then point O, and indeed all points on the horizontal diameter, will also be travelling in the direction of the arrow K at 6 in. per second. Again, at any instant during the wheel's rotation, point B is moving tangentially to the rim; at the instant it reaches the position shown, its direction is thus vertically downwards, i.e. at right angles to the horizontal radius OB. Thus the point has two velocities, one horizontally forward at 6 in. per second, and one vertically downward at 6 in. per second. These two velocities are the components of the resultant velocity required.

Draw BC parallel to OB, and make it represent 6 in. per second to a velocity scale of, say, 1 in. to 6 in. per second. Draw BD at right angles to OB, and also make it represent 6 in. per second to the same scale. Complete the parallelogram by drawing DE parallel to BC, and CE parallel to BD. Join BE. Measure BE carefully, and calculate the required resultant velocity of the point B.

$$BE = 1{\cdot}41 \text{ in.} = 8{\cdot}46 \text{ in. per second.}$$

The result may also be found by calculation. Thus BDE is a right-angled triangle, therefore

$$\begin{aligned}(BD)^2 + (DE)^2 &= (BE)^2.\\ \therefore\ 6^2 + 6^2 &= (BE)^2,\\ (BE)^2 &= 36 + 36.\\ \therefore\ BE &= \sqrt{72}\\ &= 8{\cdot}484 \text{ in. per second.}\end{aligned}$$

As shown, a slight discrepancy may appear if the length BE is measured instead of calculated.

Notice that at the instant point B is moving at 8·484 in. per second, point F is stationary (i.e. its velocity is zero), while point G is moving faster than any other point on the rim, because, for an infinitesimal period, the point G is rotating about the point F at a distance FG; all points on the rim occupy both positions, and every other position, in each complete revolution of the wheel.

Exercises, with answers, on p. 158.

CHAPTER VI

THE TRIANGLE OF FORCES

In many cases, mechanical problems involve the meeting or intersection at one point of three lines of force acting in the same plane; such problems may be solved very conveniently by the application of what is termed the TRIANGLE OF FORCES.

PROPOSITION OF THE TRIANGLE OF FORCES.—The proposition asserts that:—If any three forces, acting at a common point in one plane, are in equilibrium, and a triangle be drawn with its three sides respectively parallel to the directions of the three forces, taken in order, then the magnitudes of the three forces will be proportionate to the lengths of the three sides of the triangle.

The converse is also true, namely, that if the three sides of a triangle taken in order can be drawn parallel to the directions of and with lengths proportionate to the magnitude of three forces acting at a common point in one plane, then the three forces are in equilibrium.

The truth of the proposition may be proved by using

apparatus exactly similar to that shown in fig. 21, p. 45, with reference to the Parallelogram of Forces, by simply placing a different interpretation upon the results obtained.

Example 28.—Use weights, &c., as in Example 16, but pin a fresh sheet of paper in place of that shown at MNPQ, fig. 21.

Notice that the system is in equilibrium, and that

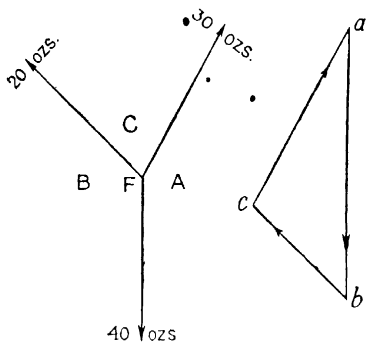

Fig. 27

three forces of 20 oz., 40 oz., and 30 oz. respectively meet at the point F. Mark point F on the paper, and three other points, one on each of the three cords, in order to obtain the directions of the forces; then remove the paper from the board.

Join these three points to the common point F, as shown in fig. 27, and mark directions and magnitudes as indicated. It will be found convenient to name the forces by lettering the spaces, the method being known as Bow's Notation. Thus, the 40-oz. force is the force AB, the line between the two letters; similarly, the 20-oz. force is the force BC, while the 30-oz. force is

the force CA. It is important that the spaces be lettered *in order*, and that the order of lettering is always in one direction, that shown being most commonly used.

Choose a force-scale of, say, 1 in. to 16 oz. Commencing with the force AB, draw *ab* parallel to the force AB, and vertically downward, making it $2\frac{1}{2}$ in. long = 40 oz.; letter it as shown, *a* at the top and *b* at the bottom, the order of the letters indicating the direction of the force. Take the next force in order, BC, and from *b* draw *bc* parallel to BC, and $1\frac{1}{4}$ in. long = 20 oz. Now take the third force, CA, and from *c* draw *ca* parallel to CA. If the work has been done accurately, the drawing of this third line should complete the triangle exactly. As a check, measure *ca*; it should be exactly $1\frac{7}{8}$ in. long, to represent 30 oz.

The left-hand illustration in fig. 27 is termed a frame diagram, and *capital* letters are invariably used. The right-hand illustration is a force or stress diagram, and *small* letters are invariably used. Rigid adherence to these rules, and to the taking of the letters and forces *in order*, will result in an almost mechanical method of obtaining solutions regarding the directions and magnitudes of these forces meeting at a point in one plane.

Notice that *ab* in the stress diagram is vertically downwards, corresponding to AB in the frame diagram. Similarly, *bc* in the stress diagram is obliquely upwards towards the left, just as is BC in the frame diagram; and again, *ca* in the stress diagram is obliquely upwards towards the right, corresponding to CA in the frame diagram. Also notice that the lengths of the three sides, *ab*, *bc*, and *ca*, are respectively $2\frac{1}{2}$ in., $1\frac{1}{4}$ in., and $1\frac{7}{8}$ in. long, representing the magnitudes of the three forces AB, BC, and CA, which are

40 oz., 20 oz., and 30 oz. respectively, to a scale of 1 in. to 16 oz.

Example 29.—A leaf in a centre-shed handloom is attached to a lifting hook by means of three cords arranged as in fig. 28. If the pull in the cord AB amounts in all to 5 lb., find the magnitude and directions of the stresses in cords BC and CA.

Make use of the sizes given to construct a frame diagram to any suitable scale. Draw *ab* parallel to AB, lettering it as shown, since the force is vertically

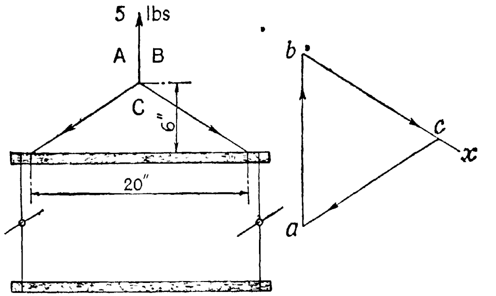

FIG. 28

upwards; make it long enough to represent 5 lb., say $2\frac{1}{2}$ in. to a scale of 1 in. to 2 lb. Now draw *bc* parallel to BC, continuing it to point *x* since the exact position of *c* is not known. The next step is to draw *ca* parallel to CA. Although point *c* is not known, the line may be drawn because point *a* is known and the direction of *ac* is parallel to AC, i.e. *ca* is parallel to CA. This obtains the triangle *abc*, which is the desired stress diagram. Measure *bc* and *ca*, and find the magnitude of the forces they represent; thus

bc = force in BC = 2·43 in. = 4·86 lb.
ca = ,, ,, CA = 2·43 in. = 4·86 ,,

The letters give the direction of the forces; these are marked by arrow-heads:

(1) On the stress diagram, *a* to *b*, *b* to *c*, *c* to *a*, as shown; and (2) on the frame diagram, being thither transferred from the stress diagram. Note that all the three arrow-heads in the frame diagram point outwards; this fact implies that the stress in the cords is tension, the kind of stress that tends to lengthen the article. It is evident that flexible material, such as twine or cord, or chain-like structures, could be used only for

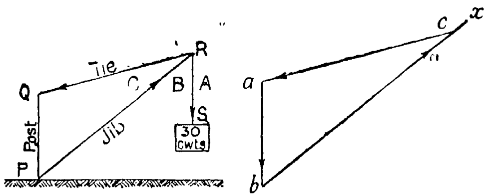

Fig. 29

tensional stresses; stiffer material must be used for compressive stresses.

Example 30.—A hand-worked jib crane is employed in a factory courtyard to raise a case of machinery weighing 30 cwt. The crane post is 12 ft. high, while the tie-rods and jib are 25 ft. and 30 ft. long respectively. Find the direction and magnitude of the stresses produced in the jib and the ties by this load.

Draw the frame diagram PQRS, fig. 29, to a scale of 1 in. to 10 ft. PQ is the crane post, QR the tie-rod, and RP the jib, while RS represents the chain carrying a load of 30 cwt. At point R, three forces meet, and since the system must be in equilibrium, the Triangle of Forces may be applied. Letter the three spaces A, B, and C as shown.

Now draw *ab* parallel to AB, and make it 1½ in. long, to represent 30 cwt., to a scale of 1 in. to 20 cwt.; letter it *ab* as shown—downwards—since the load acts downwards. Taking the next force in order, from *b* draw *bc* parallel to BC, continuing it to *x* since point *c* is not definitely fixed. From *a* draw *ac* parallel to CA, and complete the triangle. Measure *bc* and *ca* in order to find the magnitudes; thus

$$bc = \text{stress in jib} = 3{\cdot}08 \text{ in.} = 61{\cdot}6 \text{ cwt.}$$
$$ca = \text{stress in tie} = 3{\cdot}72 \text{ in.} = 74{\cdot}4 \text{ cwt.}$$

The letters give the directions, the method being similar to that shown in the previous example. The forces in AB, the chain, and in CA, the tie-rod, act *away* from the point; consequently these members are in tension. The force in BC, the jib, acts *towards* the point; therefore this member is in compression.

Example 31.—A rectangular trap-door is made in the roof of a factory loading dock, in order that cloth and yarn may be conveniently loaded on to lorries or cars underneath. The trap-door measures 5 ft. square and weighs 100 lb. It is supported in its horizontal position by a chain, as indicated in fig. 30, at a point on its outer edge reaching to a second point 7 ft. vertically above the hinge. By means of a stress diagram, determine the tension in the chain, and the reaction on the hinges.

After the frame diagram has been drawn, this problem, at first sight, does not seem capable of solution by the application of the principle involved in the Triangle of Forces. The weight of the door may be taken as acting at its centre of gravity, i.e. vertically downwards from the centre of the door, as indicated by the arrow-head D. There are, therefore, only three forces acting on the door, and, since the

whole system is in equilibrium, the lines of action of these three forces must either pass through one point, or they must be parallel to each other. This latter supposition is, obviously, wrong, since the direction of the stress in the chain cannot be parallel to the

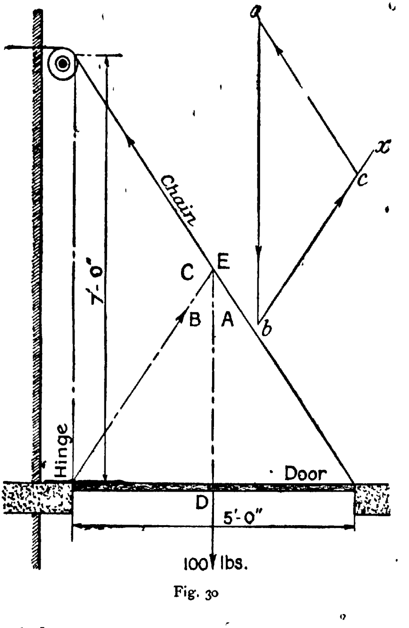

Fig. 30

downward force at D; hence, the three forces must meet at a common point. Now a force will produce the same effect at whatever point in its line of direction it may be assumed to act (see p. 36); so that the force-line D may be produced upwards until it intersects the line of action in the chain at the point E. Again, because the system is in equilibrium, the line

of the force-action in the hinge must pass through this same point; these two lines are shown dotted in fig. 30. Point E is therefore the point required, and the spaces round it may be lettered in the usual way.

When this stage is reached, the solution may be obtained on the same lines as in the previous two examples; the resulting stress diagram is then *abc*, fig. 30, *ab* being made to represent 100 lb. to a scale of 1 in. to 40 lb.

bc = stress on hinge = 1·53 in. = 61·2 lb.,
ca = stress in chain = 1·53 in. = 61·2 lb.

The stress in *bc* is *towards* the point, so that there is a compressive stress or thrust on the hinge; the stress in CA is *away from* the point, so that the chain is in tension, as might easily be deduced from the mere fact that a chain is so employed.

The methods employed above will only give accurate results if the drawings themselves are made accurately, and to a scale as large as possible.

The propositions of the Parallelogram of Forces and the Triangle of Forces show how to deal with cases when two and three forces respectively meet at a point. The truth of a third proposition may also be established—that of the POLYGON OF FORCES, which asserts that if *any* number of forces in one plane act upon a body and that body is in equilibrium, then the forces must pass through one point or be parallel; if they pass through one point, they may be represented in direction and in magnitude by the sides (taken in order) of the polygon. As might naturally be expected, the Polygon of Forces is only of use in comparatively complicated cases, and its further consideration must be deferred for the present.

Exercises, with answers, on p. 159.

CHAPTER VII

MOMENTS

MOMENT OF A FORCE.—Suppose any body, such as that shown diagrammatically in fig. 31, is resting on a surface containing the point P, and that a force F is applied to it in the direction FA, as indicated by the arrow-heads. Obviously, this force F would either turn, or tend to turn, the body A about the point P. It is necessary in many cases to be able to measure or calculate this tendency to turn, whether motion is actually caused or not.

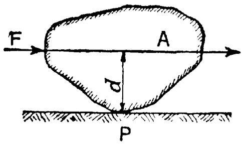

FIG. 31

The tendency of the force to cause the body to move about the point P is called the MOMENT of the force, and it may be defined as follows:—The moment of a force, with respect to any given point, is the product of the force and the perpendicular distance between the point and the line of action of the force. Referring again to fig. 31, if F is the force in pounds, FA the line of action of the force, and P the given point, the moment of the force F is $F \times d$, where d is the perpendicular distance between the given point P and the line of force-action FA.

The units used in the measurement of moments are those of force and distance. If the force is in pounds, and the distance in inches, the value of the moment is in pound-inches or lb.-in.; if the force is in tons and the distance in feet, the moment is in ton-feet or ton-ft.

One must be careful to distinguish between the compound units of moments and those of work done. The former are expressed in pound-inch, ton-foot, &c., where the force unit (pound, ton, &c.) is made to *precede* the distance unit (inch, foot, &c.); the value of work is invariably expressed by such units as foot-pound, foot-ton, &c., where the force unit always *follows* the distance unit.

As a concrete example, assume that a uniform bar AB, fig. 32, supports loads or downward forces of 5 lb. and 75 lb. at its left- and right-hand ends

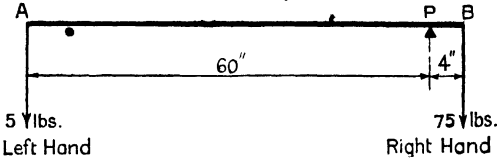

Fig. 32

respectively. If it is found by trial that the bar can be balanced about the point P, the distances AP and BP being 60 in. and 4 in. respectively, then the moment of the left-hand force A about the point P is

$$5 \text{ lb.} \times 60 \text{ in.} = 300 \text{ lb.-in.}$$

The moment of the right-hand force B about the point P is

$$75 \text{ lb.} \times 4 \text{ in.} = 300 \text{ lb.-in.}$$

The moments on either side of the point P are thus equal in magnitude; it will be shown shortly that this condition is necessary for equilibrium. The *directions* of the moments are not similar. The moment on the side B tends to turn the bar about

the point P in the same direction as the hands of a clock move, i.e. clockwise. On the other hand, the moment on the side A tends to turn the bar about the point P in the opposite direction, i.e. counter-clockwise. Thus the moments on either side of point P are equal in magnitude but opposite in direction; this is the full condition necessary for equilibrium.

THE PRINCIPLE OF MOMENTS.—If any number of forces act in one plane on a rigid body, and if that body is in equilibrium, then the sum of the clockwise moments about any point is equal to the sum of the counter-clockwise moments about the same point.

A second definition is:—If the clockwise moments be termed negative (−), and the counter-clockwise moments be termed positive (+), the Principle of Moments asserts that the algebraic sum of the moments of the forces acting in one plane about a point in a body in equilibrium is *zero.*

The Law or Principle of Moments is often applied to the solution of problems dealing with all forms of levers, and hence it is often called the "Law of the Lever". It will be convenient to define just what constitutes a lever, and then to show how the Principle of Moments asserted above may be proved by applying it to a simple lever or its equivalent.

THE LEVER.—A machine may be defined as an assemblage or arrangement of fixed and moving parts constructed for the transmission of force and motion, and capable of modifying or constraining the force or motion thus transmitted.

For example, a steam-engine, such as is used for driving a textile factory, is a machine in which the expansive property of steam at high pressure is constrained to act on a piston in a cylinder in such a

manner as to cause the piston to move backwards and forwards in a straight-line motion. This straight-line or rectilinear motion is modified by the use of a connecting rod and a crank into circular or rotary motion. The rotary motion of the crank may cause a fly-wheel to revolve, and ropes on the fly-wheel will transmit the force and motion thus obtained from the engine to any other assemblage of movable parts, and cause them to be set in motion also. The steam-engine might be used, for example, to drive a number of spinning mules. The rotary motion of the engine would be transmitted by shafting, belting, gears, &c., to the pulley of the spinning mule, itself simply another form of machine, and there the power of the prime mover would be constrained and modified to draw out the cotton rove, twist it into a yarn, and coil it upon a spindle into the well-known cop shape.

The *Lever* is nothing more nor less than a simple or elementary machine; it is, indeed, one of the so-called six mechanical elements or simple machines, the others being the wheel and simple axle, the wheel and compound axle, the pulley, the inclined plane, and the screw. It will be shown, however, that the screw may be regarded as a special form of the inclined plane, and that the various forms of wheel and axle and pulley may be regarded as special forms of the lever.

A lever may be defined as a rigid piece capable of turning about a point known as the fulcrum or pivot, and in which there are two or more points where forces may be applied; it is used, as indicated above, for transmitting and modifying force and motion. In textile machinery, it generally takes the form of a stiff metal bar or rod, such as the treadle of a loom,

as in fig. 37, the picking shaft of a loom, or the lever of a roving frame pressing roller, such as is shown in fig. 2. At other times, however, levers of exceedingly delicate structure are employed; nevertheless, heavy, intermediate, and light types come under the same principle.

Example 32.—To prove the Principle of Moments when applied to the case of a lever in equilibrium.

The apparatus required need not be elaborate. Obtain a rigid wooden bar to serve as a lever; drill a small hole through its geometrical centre, and suspend it by a loop of fine string to see that it balances horizontally. If it is unbalanced, add small weights of any kind to the lighter side until it is perfectly balanced about the central hole. The rod will serve its present purpose better if the near or the upper face is marked off into inches and tenths of an inch; this can be conveniently done by pasting on strips of 10 × 10 squared paper. A stand will also be necessary from which the lever may be freely suspended, and a series of known weights should be available. The suggested complete apparatus is illustrated in fig. 33.

After the rod or lever has assumed an absolutely horizontal position, take any two weights, equal or unequal, and hang them by means of loops of fine cord on opposite sides of the fulcrum F. The weight P on the left-hand side of the fulcrum exerts on the bar a counter-clockwise moment, tending to turn the bar or lever about the point F; the moment of this force is the product of the weight P and the distance p. This moment is balanced by the clockwise moment of the weight W placed at any suitable distance w, the moment being $W \times w$ or Ww. Six different readings should be taken, choosing different values

of P and W, and of p and w, after which the respective moments could be calculated. The whole

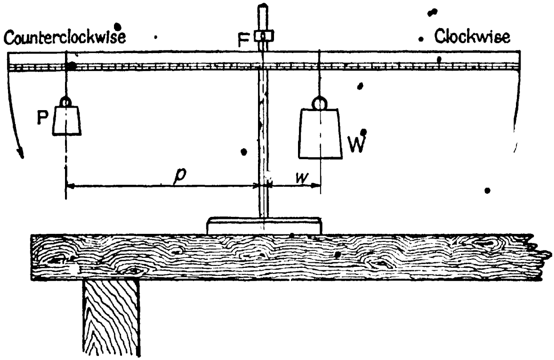

Fig. 33

may be conveniently tabulated as under, the figures given being those obtained by actual experiments.

TABLE I

Counter-clockwise.			Clockwise.		
P	p	P × p	W × w	w	W
7	13·8	96·6	96·5	19·3	5
6	13·8	82·8	83·0	16·6	5
5	12·2	61·0	61·2	15·3	4
4	16·5	66·0	66·0	16·5	4
3	9·7	29·1	29·2	14·6	2
2	14·7	29·4	29·4	4·2	7

All the above weights are in pounds, and the dis-

tances in inches; the moments are therefore in pound-inches.

From the results of the above experiments it may be inferred:—

1. That the Principle of Moments as defined above is correct, since the counter-clockwise moment Pp is equal to the clockwise moment Ww, i.e. Pp = Ww. The slight inaccuracies shown in the table are due solely to experimental errors, and mainly to inexact measurements of the distances p and w.

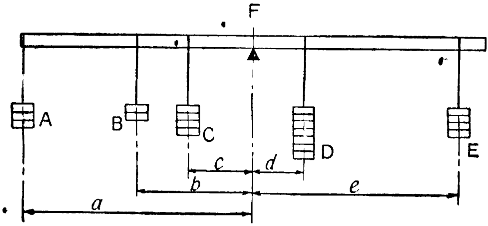

Fig. 34

2. That if the forces are equal, the distances also are equal. (See the 4th line in table, p. 71.)

It might also be inferred that the number of forces on either side of the fulcrum should not affect the principle; this phase, however, may be brought out more comprehensibly by a second example.

Example 33.—Verify the Principle of Moments by using a series of loads or weights on each side of the fulcrum.

The apparatus used in connection with fig. 33 may be used for the present experiment, but the weights are preferably arranged as in fig. 34, and the results tabulated as shown on following page.

The following numbers or values are again those obtained by actual experiments.

TABLE II

Counter-clockwise.			Clockwise.		
Moment.	Values.	Moment in lb.-in.	Moment in lb.-in.	Values.	Moment.
A × *a* B × *b* C × *c*	3 × 18 2 × 9 4 × 5	54 18 20	28 64	7 × 4 4 × 16	D × *d* E × *e*
Sum of Moments = 92			92 = Sum of Moments		

It is advisable to note that it is the *direction* of the moment that is important, and not so much the side of the lever upon which the force acts, or whether the force is a pull or a push. In the examples illustrated in figs. 33 and 34, all the left-hand moments are counter-clockwise moments, simply because all the forces act downwards; it would be quite easy to produce a clockwise moment on this left-hand side of the lever by causing an *upward* force to act on the lever.

MOMENTS OF LEVERS.—There are three classes or orders of levers, the various orders being distinguished by the relative positions of the force (pull or push) P, the load, weight, or resistance W, and the pivot or fulcrum F.

1. In fig. 35 the fulcrum F is between the force P and the resistance W. The diagrammatic view at A shows a "skeleton" arrangement of this order, while the drawing at B is a typical example of the parts employed for a drag-lever in a loom fitted with an ordinary negative let-off for "pace" motion. In this example an upward or downward pull applied at P acts at a certain distance from the fulcrum F, and

balances an upward or downward resistance W. Such levers are commonly known as two-armed levers, often absolutely straight, but at other times curved to meet

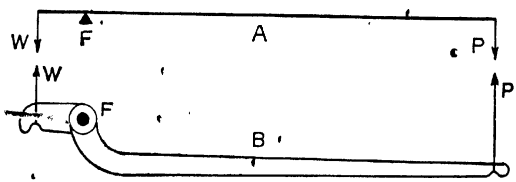

Fig. 35

the disposition of other parts in a machine. For example, the long arm in B is curved near the fulcrum F; the effective length of this arm is the straight line

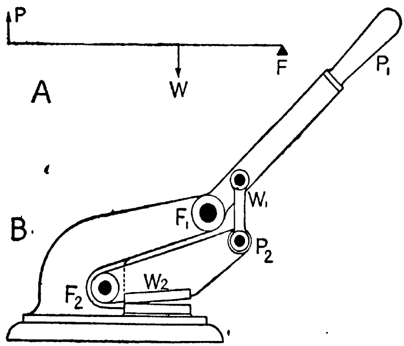

Fig. 36

from the fulcrum to the point of application of the force P. A double lever of this order is exemplified by a pair of ordinary scissors.

2. The resistance W is between the force P and the

fulcrum F, as shown diagrammatically at A, fig. 36. The application of the principle in practice is demonstrated by the hand-press at B, which is used for shearing off hoops intended for binding securely hydraulic-press-packed bales of yarn, cloth, and the like. (See *The Finishing of Jute and Linen Fabrics*, by T. Woodhouse, pp. 119 to 141.) Thus, a force applied at P, diagram A, fig. 36, overcomes a resistance W, each acting at its particular distance from the fulcrum F. The press or hoop-cutter shown at B provides an example of a compound lever of this order, where

P_1 and P_2 are the forces;
W_1 ,, W_2 ,, resistances; and
F_1 ,, F_2 ,, fulcra.

A very common example of the double type is a pair of nut-crackers.

3. The force P is between the fulcrum F and the resistance W as in fig. 37. Here also A is the

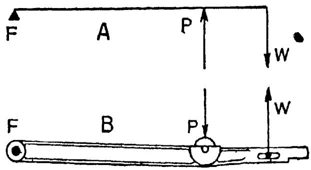

Fig. 37

skeleton arrangement, while B is an actual example of a treadle lever in an ordinary tappet or wyper power loom. In each case a force applied at P overcomes a resistance at W, the conditions for equilibrium being according to the Principle of Moments, viz. that the sum of the clockwise and counter-clockwise moments

acting on the lever be zero. Another common example of this order of lever is the steam safety-valve, used on boilers and on warp-dressing machines and dyehouse steam-pipes. A double lever of this order is exemplified by a pair of tongs.

The Law of the Lever may therefore be stated as under:

~~Let~~ P be the force (push or pull) acting on the lever, and p be the distance between the line of action of the force and the fulcrum; also, let W be the weight, load, or resistance overcome, and w the distance of its line of action from the fulcrum: then, in every case,

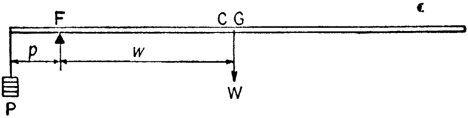

Fig. 38

irrespective of the class of lever, the Principle of Moments enables it to be asserted that

$$P \times p = W \times w, \text{ or } Pp = Ww.$$

Example 34.—To find, by experimental means, the approximate weight of a uniform wooden bar or lever.

The apparatus required consists of a wooden bar, say 72 in. × 1¼ in. × ½ in., and a known weight, say 4 lb. or 64 oz. The parts are arranged as illustrated in fig. 38; the known weight of 4 lb., indicated by the letter P, is hung over one end of the bar, and the bar is moved about until a position with respect to the chosen fulcrum F is obtained, which gives perfect equilibrium. When this position has been found, the bar is in equilibrium under the action of two moments,

equal in magnitude but opposite in direction. The counter-clockwise moment is due to the known weight P acting at the distance p from the fulcrum F, while the clockwise moment is due to the unknown weight of the bar or lever W acting at its centre of gravity, and at a distance w from the fulcrum F. Equilibrium should be established when

$$Pp = Ww.$$

Measure the distances p and w; with these and the known weight P, 64 oz., calculate the weight of the bar. The following values are recorded from an actual experiment:

$$P = 4 \text{ lb.}; \; p = 8 \text{ in.}; \text{ and } w = 28 \text{ in.}$$

By the principle of Moments as above:

$$\begin{aligned} Pp &= Ww, \\ \text{whence } W &= \frac{Pp}{w}, \\ \text{hence } W &= \frac{4 \text{ lb.} \times 8 \text{ in.}}{28 \text{ in.}} \\ &= \tfrac{8}{7} = 1\tfrac{1}{7} \text{ lb.} \end{aligned}$$

The actual weight of the bar as obtained by the usual method of weighing was 18¼ oz., so that the experimental error is very small.

Confirmation of the above method of finding the weight of a uniform rod is as follows:—

Example 35.—A 72-in. bar, weighing 18¼ oz., is fulcrumed at a point 8 in. from the end; find the force P at the short end essential to maintain equilibrium.

The centres of gravity of the two overhanging parts of the bar will be midway between the fulctum and

the ends of the bar, i.e. $\frac{8 \text{ in.}}{2} = 4$, and $\frac{64 \text{ in.}}{2} = 32$;

$$\begin{aligned}
\text{hence } p &= 8 \text{ in.} \\
p_1 &= \frac{8 \text{ in.}}{2} = 4 \text{ in.} \\
P_1 &= \frac{8}{72} \text{ of } 18\tfrac{1}{4} \text{ oz.} = 2\tfrac{1}{36} \text{ oz.} \\
W &= \frac{64}{72} \text{ of } 18\tfrac{1}{4} \text{ oz.} = 16\tfrac{8}{36} \text{ oz.} \\
w &= 32 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
Pp + P_1p_1 &= Ww \\
P &= \frac{Ww - P_1p_1}{p} \\
&= \frac{16\frac{8}{36} \times 32 - 2\frac{1}{36} \times 4}{8} \\
&= \frac{519\frac{1}{9} - 8\frac{1}{9}}{8} = \frac{511}{8}. \\
\therefore P &= 63\tfrac{7}{8} \text{ oz.}
\end{aligned}$$

If, therefore, the known weight in Example 34 had been $63\frac{7}{8}$ oz. instead of a 4-lb. or 64-oz. weight, the moments considered would have been

$$\begin{aligned}
8 \times 63\tfrac{7}{8} &= 28 \times 18\tfrac{1}{4} \\
511 &= 511.
\end{aligned}$$

With the 4-lb. or 64-oz. weight, the actual distances would have been

$$p = 7{\cdot}98785 \text{ in., and } w = 28{\cdot}01215 \text{ in.,}$$

obtainable only in practice by the use of a Vernier Scale, or better still by a bar micrometer.

Example 36.—A uniform straight metal bar (fig. 39) weighs 16 lb. and is 4 ft. long. It is fulcrumed at one end, and carries a load of 10 lb. at a point 12 in. from

the fulcrum. Find the upward force at the free end which will keep the bar horizontal, i.e. in equilibrium.

By the principle of moments, when the bar is in equilibrium, the sum of the clockwise moments must

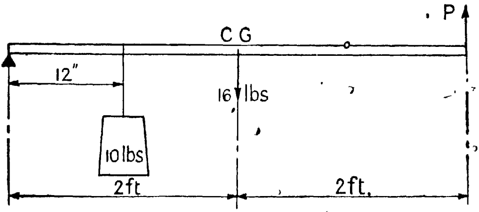

Fig. 39

equal the sum of the counter-clockwise moments; that is:

$$\text{Clockwise} = \text{Counter-clockwise.}$$
$$(16\text{ lb.} \times 2\text{ ft.}) + (10\text{ lb.} \times 1\text{ ft.}) = P\text{ lb.} \times 4\text{ ft.}$$
$$32\text{ lb.-ft.} + 10\text{ lb.-ft.} = 4\,P\text{ lb.-ft.}$$
$$\text{i.e. } 4\,P = 42;$$
$$\text{whence } P = \frac{42}{4} = 10\tfrac{1}{2}\text{ lb.}$$

Example 37.—An iron lever, fig. 40, of uniform section is 20 in. long and weighs 8 lb., and a weight of 12 lb. is supported at one end. Find the fulcrum or point upon which the lever will remain in equilibrium, i.e. the point about which the lever will balance.

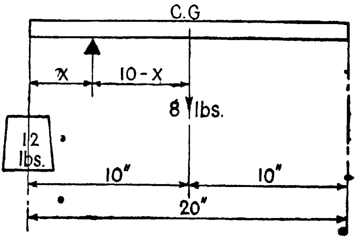

Fig. 40

Let the distance between the fulcrum and the 12-lb. weight be x in., then the distance between the fulcrum and the centre of gravity of the lever must be $(10 - x)$ in.; consequently:—

Counter-clockwise moments = clockwise moments,

i.e.,

$$12 \times x = 8\,(10 - x),$$
$$12\,x = 80 - 8x,$$
$$20\,x = 80.$$
$$\therefore \quad x = 4 \text{ in.}$$
$$\text{and } (10 - x) = 6 \text{ in.},$$

so that the bar will balance at a point which is 4 in. from the 12-lb. weight, and 6 in. from the centre of gravity of the lever.

Example 38.—The safety-valve of a boiler, of the type illustrated in fig. 41, is required to allow steam

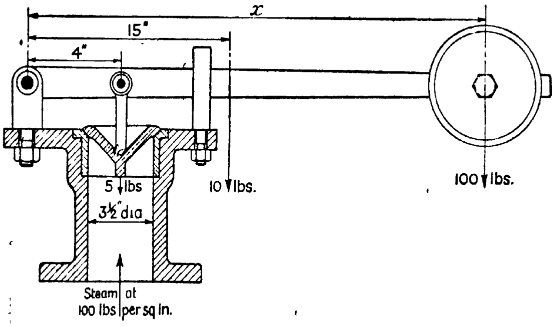

Fig. 41

to escape at a gauge pressure of 100 lb. per square inch. Find where the weight should be placed on the lever.

The total upward force tending to lift the valve from its seat is due to the steam pressure of 100 lb. per

square inch on the projected area of the valve, a circle of $3\frac{1}{2}$ in. diameter. This upward pressure is balanced partly by the weight of the valve itself, and partly by the moments exerted by the weight and the weight of the lever, all tending to keep the valve down upon its seat.

N.B.—The dimensions given in fig. 41 are not to scale.

Total upward force on valve

$$\begin{aligned} &= \text{Valve area} \times \text{pressure in pounds per square inch} \\ &= (\cdot 7854 \times 3\cdot 5^2) \text{ sq. in.} \times 100 \text{ lb. per square inch} \\ &= \cdot 7854 \times 3\cdot 5 \times 3\cdot 5 \times 100 \\ &= 962\cdot 115, \text{ say } 962\cdot 5 \text{ lb.} \end{aligned}$$

But the valve itself *directly* balances 5 lb. of this force, so that the total effective force is

$$962\cdot 5 \text{ lb.} - 5 \text{ lb.} = 957\cdot 5 \text{ lb.}$$

Then, by the Law of the Lever,

$$\begin{aligned} \text{Clockwise moments} &= \text{Counter-clockwise moments.} \\ (100 \text{ lb.} \times x \text{ in.}) + (10 \text{ lb.} \times 15 \text{ in.}) &= 957\cdot 5 \text{ lb.} \times 4 \text{ in.} \\ 100x \text{ lb.-in.} + 150 \text{ lb.-in.} &= 3830 \text{ lb.-in.} \\ 100x &= 3830 - 150. \\ \therefore x &= \frac{3680}{100} \\ &= 36\cdot 8 \text{ in.} \end{aligned}$$

In order, therefore, that steam should blow off at 100 lb. per square inch pressure, the weight should be placed at a distance of 36·8 in. from the fulcrum of the lever.

Example 39.—In a roving frame pressure is applied

to the pressing roller and the drawing roller, see fig. 2, as indicated. The strap S is between two pressing rollers each $1\frac{3}{8}$ in. wide, and the weight W is $12\frac{3}{4}$ lb. The distance between the fulcrum F of the lever L and the weight W is 15 in., and that between the fulcrum and the point at which the pressure is applied is $1\frac{1}{4}$ in. Find the pressure on the rollers per inch of width.

$$\text{Total pressure} = 12\tfrac{3}{4}\text{ lb.} \times \frac{15\text{ in.}}{1\frac{1}{4}\text{ in.}}$$
$$= \frac{51}{4} \times \frac{15}{1} \times \frac{4}{5} = 153\text{ lb.}$$

Each roller is $1\frac{3}{8}$ in. wide, i.e. the total pressure of 153 lb. is distributed over ($2 \times 1\frac{3}{8}$ in.) or $2\frac{3}{4}$ in.

$$\therefore \text{Pressure per inch width} = \frac{153\text{ lb.}}{2\frac{3}{4}\text{ in.}}$$
$$\doteqdot 55{\cdot}5\text{ lb.};$$

or, shortly:

$$\frac{12\frac{3}{4}\text{ lb.} \times 15\text{ in.}}{1\frac{1}{4}\text{ in.} \times (2 \times 1\frac{3}{8}\text{ in.})} \doteqdot 55{\cdot}5\text{ lb. pressure per inch width of roller.}$$

Example 40.—In a certain flax spreader, pressure is applied to the boss or front rollers by means of a system of compound levers and weights, a diagram of which appears at A, fig. 42, while the actual arrangement of the parts is illustrated at B in the same figure. The pressure is applied on each side of a double roller R, each face being $7\frac{1}{2}$ in. wide. Find the pressure per inch width. To simplify matters, it may be taken that one weight and set of levers act on each width.

This type of problem may be worked out in parts, or in one complete formula. Note that the studs, or fulcra, S and T, fig 42, are fixtures in the framing of the machine.

$$\text{Pull on link L} = 15 \text{ lb.} \times \frac{21 \text{ in.}}{3 \text{ in.}}$$
$$= 105 \text{ lb.}$$
$$\text{Pull on rollers R} = 105 \times \frac{30}{2}$$
$$= 1575 \text{ lb.};$$
$$\text{and } \frac{1575 \text{ lb.}}{7\frac{1}{2} \text{ in.}} = 210 \text{ lb. per inch width of roller;}$$

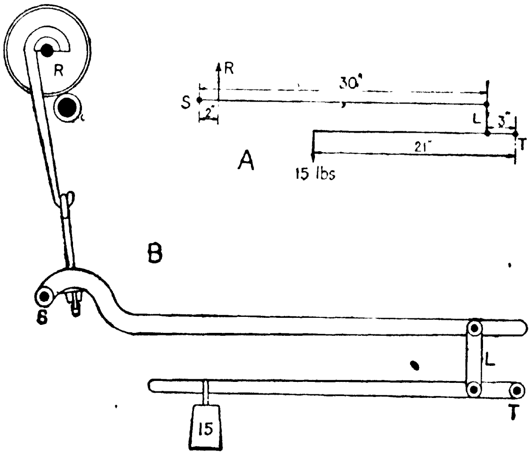

Fig. 42

or, shortly:

Pull or pressure on rollers in pounds per inch width

$$= 15 \times \frac{21}{3} \times \frac{30}{2} \times \frac{1}{7\frac{1}{2}}$$
$$= 210 \text{ lb. per inch width.}$$

Exercises, with answers, on p. 160.

CHAPTER VIII

REACTIONS AT SUPPORTS
PARALLEL FORCES AND COUPLES

In a previous section, dealing with resultants, it is stated that if a body is in equilibrium under the action of any system of forces, then either the force lines meet at a common point, or they are parallel. The former case has already been discussed; the latter is now to be dealt with briefly. In either case, since the body *is* in equilibrium, the algebraic sum of the force moments must be zero.

The latter case, where the forces are acting in parallel lines, finds a useful application in questions

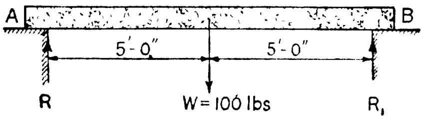

Fig. 43

affecting beams of all kinds. It is appropriate, therefore, that a beam should be chosen as an illustrative example; and it will be understood that the rails forming the supports of textile machinery of all kinds are, in reality, beams of the nature of those about to be described.

Example 41.—Let AB, fig. 43, represent a beam which is 10 ft. between its two points of support and weighs 100 lb. It is obvious that the weight W of the beam may be considered as a vertically downward force of 100 lb. acting through the centre of gravity of the beam; it will be equally obvious that this downward force must have an equal and opposite

reaction. The reaction in this case is in two parts, since the beam is supported at two points. A little consideration of the stable condition of the beam should indicate that the two reactions must be vertically upward, and that they will be equal in magnitude, i.e. each 50 lb. The beam is thus a body in equilibrium under the action of three parallel forces, one vertically downwards of 100 lb., and two vertically upwards of 50 lb. each:

$$50 \text{ lb.} + 50 \text{ lb.} = 100 \text{ lb.}$$

The reactions at the supports of the beams may be calculated by an application of the Principle of Moments. Suppose that through any cause the beam in fig. 43 could be made free to rotate from its horizontal position, say about the point A. Then, since the beam is in equilibrium, the sum of the clockwise moments must equal the sum of the counter-clockwise moments.

The clockwise moment is

$$100 \text{ lb.} \times 5 \text{ ft.} = 500 \text{ lb.-ft.}$$

The counter-clockwise moment is

$$R_1 \text{ lb.} \times 10 \text{ ft.} = 10\, R_1 \text{ lb.-ft.},$$

so that $10\, R_1 \text{ lb.-ft.} = 500 \text{ lb.-ft.},$

whence $R_1 = 50 \text{ lb.}$

In a similar manner, by supposing the beam free to rotate about point B, it will be found that

$$R = 50 \text{ lb.}$$

Both these results would be anticipated from the nature of the case; the essential point to notice is that the results have been obtained by applying the Principle of Moments. It will be found that the Principle of Moments applies in every case of parallel forces, no

matter how complicated the system may appear. In all cases the *modus operandi* is similar, viz. in order to find the reaction at one support, take moments about the other. In the above case, to find R_1, moments are taken about R (or A), the force R itself being neglected, since, its distance from A is 0, its moment about A is also 0.

The student should endeavour to prove the truth of the method of applying the Principle of Moments to

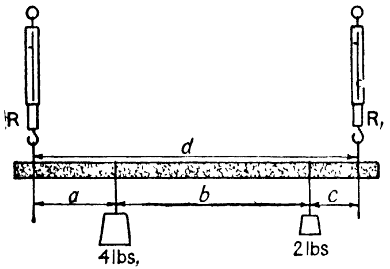

Fig. 44

find the reactions at the ends of a loaded horizontal beam by carrying out for himself an experiment on the lines indicated below.

Example 42.—To find the reactions at the supports of a horizontal beam when loaded, say, with weights of 2 lb. and 4 lb. at two points between the supports.

The apparatus consists of a wooden bar similar to that used in Example 34, two light spring balances, and two weights of 2 lb. and 4 lb. respectively. It will also be necessary to have a frame or stand from which the two balances and the bar can be suspended. The complete arrangement will appear somewhat as shown in fig. 44. The exercise may be conveniently

divided into 3 stages: (1) Experimental; (2) Calculation by applying the Principle of Moments; and (3) Checking results.

1. *Experimental.*—Arrange the apparatus as in fig. 44. Note the reading on each balance before placing the two weights in position; these give the reactions at the supports due to the weight of the beam alone. If the beam is horizontal, and of homogeneous material, the readings on the two balances will be identical; they should be written down, along with their sum, which is the weight of the beam.

Now place the two weights, say of 4 lb. and 2 lb., on the beam, and in any convenient positions, such as those indicated in fig. 44. Measure carefully the distances *a*, *b*, *c*, and *d*, and note the reactions R and R_1 as recorded by the spring balances. Arrange these results as shown below, the figures used being those of an actual experiment.

Before loading:

R	=	$2\frac{1}{4}$ oz.	=	reaction at left-hand support.
R_1	=	$2\frac{1}{4}$ oz.	=	reaction at right-hand support.
W	=	$4\frac{1}{2}$ oz.	=	weight of beam.

After loading:

a	*b*	*c*	*d*
$9\frac{5}{8}$ in.	$23\frac{3}{16}$ in.	$5\frac{3}{16}$ in.	38 in.

Note that $a + b + c = d$.

R	=	3 lb. 6 oz.	=	reaction at left-hand support.
R_1	=	2 lb. 14 oz.	=	reaction at right-hand support.
		6 lb. 4 oz.	=	sum of all loads acting at supports.

$$\text{Actual sum of all loads} = 4 \text{ lb.} + 2 \text{ lb.} + 4\tfrac{1}{2} \text{ oz.}, = 6 \text{ lb. } 4\tfrac{1}{2} \text{ oz.}$$

The experimental error is thus $\frac{1}{2}$ oz. in 6 lb. $4\frac{1}{2}$ oz.,

$$\text{hence } \frac{\frac{1}{2} \text{ oz.}}{6 \text{ lb. } 4\frac{1}{2} \text{ oz.}} \text{ or } \frac{\cdot 5}{100\cdot 5 \text{ oz.}} \times 100 = \text{less than } \tfrac{1}{2}\%.$$

2. *Calculation.*—To obtain the reaction at R take moments about R_1; thus:

$$\begin{aligned} 38R &= 32 \text{ oz.} \times c + 64 \text{ oz.} \times (b + c) + 4\tfrac{1}{2} \text{ oz.} \times \frac{d}{2} \\ &= 32 \times 5\tfrac{3}{16} + 64 \times 28\tfrac{3}{8} + 4\tfrac{1}{2} \times 19 \\ &= 32 \times 5\cdot 1875 + 64 \times 28\cdot 375 + 4\cdot 5 \times 19 \\ &= 166 + 1816 + 85\cdot 5 = 2067\cdot 5 \text{ oz.-in.} \end{aligned}$$

$$\begin{aligned} \therefore R &= \frac{2067\cdot 5 \text{ oz.-in.}}{38} = 54\cdot 408 \text{ oz.} \\ &= 3 \text{ lb. } 6\cdot 408 \text{ oz.} \end{aligned}$$

To obtain the reaction at R_1, take moments about R as under:

$$\begin{aligned} 38R_1 &= 64 \text{ oz.} \times a + 32 \text{ oz.} \times (a + b) + 4\tfrac{1}{2} \text{ oz.} \times \frac{d}{2} \\ &= 64 \times 9\tfrac{5}{8} + 32 \times 32\tfrac{13}{16} + 4\tfrac{1}{2} \times 19 \\ &= 64 \times 9\cdot 625 + 32 \times 32\cdot 8125 + 4\cdot 5 \times 19 \\ &= 616 + 1050 + 85\cdot 5 = 1751\cdot 5 \text{ oz.-in.} \end{aligned}$$

$$\begin{aligned} \therefore R_1 &= \frac{1751\cdot 5 \text{ oz.-in.}}{38} = 46\cdot 092 \text{ oz.} \\ &= 2 \text{ lb. } 14\cdot 092 \text{ oz.} \end{aligned}$$

Consequently

$$\begin{aligned} R + R_1 &= 3 \text{ lb. } 6\cdot 408 \text{ oz.} + 2 \text{ lb. } 14\cdot 092 \text{ oz.} \\ &= 6 \text{ lb. } 4\cdot 5 \text{ oz.} \end{aligned}$$

3. *Checking Results.*—Possibly the most orthodox method of checking such an experiment as the above would be to draw carefully frame and stress diagrams; the latter involves in the present case the use of the funicular polygon, which has not yet been considered.

It may be sufficient, therefore, to check the work by repeating the experiment with different loads and distances, and afterwards tabulate the results for comparison, as exemplified below:—

TABLE III

Reactions.	Result obtained by Experiment.	Result obtained by Calculation.	Actual Weights of the 3 articles.
			Beam, 0 lb. $4\frac{1}{2}$ oz.
R	3 lb. 6 oz.	3 lb. 6·408 oz.	1st Load, 4 lb. 0 oz.
R_1	2 lb. 14 oz.	2 lb. 14·092 oz.	2nd ,, 2 lb. 0 oz.
$R+R_1$	6 lb. 4 oz.	6 lb. 4·5 oz.	Total, 6 lb. $4\frac{1}{2}$ oz.

The following further examples illustrate different aspects of the application of the Principle of Moments to the solution of various types of problems.

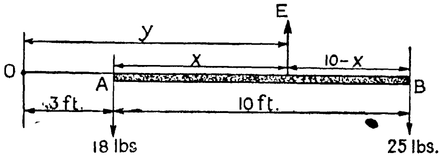

Fig. 45

Example 43.—Two forces, of 18 lb. and 25 lb. respectively, act on a 10-ft. bar as indicated in fig. 45. Find the magnitude, direction, and position of the equilibrant.

Since both forces are acting vertically downwards, they could be replaced by a single vertically downward force (the resultant) of *18 lb. + 25 lb. = 43 lb. The equilibrant is equal in magnitude, but oppositely directed to the resultant; it is therefore a force of 43 lb. acting vertically upwards.*

Let the *position* of the equilibrant be x ft. from A; it is then $(10 - x)$ ft. from B, as shown at E. Hence, taking moments about A, we have:

$$43\,x = 25 \times 10$$

$$\text{whence } x = \frac{25 \times 10}{43} = 5{\cdot}814 \text{ ft.},$$

i.e. the equilibrant must act at a point 5·814 ft. distant from A. But it might not yet be clear whether the distance is 5·814 feet to the left of A, or a similar distance to the right of A. To ascertain this, notice that the distance between E and B is $(10 - x)$ ft., so that the point E is $(10 - 5{\cdot}814)$ ft. $= 4{\cdot}186$ ft. from B. Point E is therefore 5·814 ft. from A, and 4·186 ft. from B; it is therefore as indicated in fig. 45, this being the only point that satisfies both requirements.

It is not absolutely essential to take moments about either point A or point B. Any point outside the bar, such as O, may be selected. Thus, assume that the point O is 3 ft. to the left of A, and that the distance between O and E is y ft.; then, taking moments about O we shall find that:

$$\begin{aligned} 43y &= (18 \times 3) + (25 \times 13) \\ &= 54 + 325 \\ &= 379. \\ \therefore y &= \frac{379}{43} = 8{\cdot}814 \text{ ft.} \end{aligned}$$

This result confirms the one already found, since the position is:

$$8{\cdot}814 \text{ ft.} - 3 \text{ ft.} = 5{\cdot}814 \text{ ft. from the end A,}$$

i.e. $y - 3$ ft. $= x$.

Example 44.—A cast-iron lever in a textile machine is to be of uniform section, and to weigh 40 lb. It

has to support a load of 20 lb. at one end, and must balance at a point 18 in. from this end. Find the length of the lever.

A diagrammatic view of the problem is given in fig. 46. Let O be the centre of the lever AC, and x be the distance between O and the fulcrum F. The length of the lever is thus $2(x + 18)$, and the solution

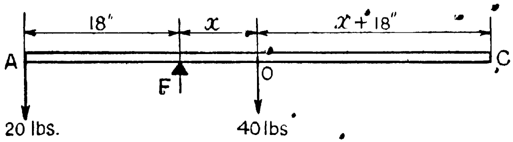

Fig. 46

of the problem demands the finding of the proper value of x.

By the principle of moments—

$$40x = 20 \times 18.$$

$$\therefore x = \frac{20 \times 18}{40} = 9 \text{ in.}$$

$$\begin{aligned} \text{Length of lever} &= 2(x + 18) \\ &= 2(9 + 18) \\ &= 2 \times 27 \\ &= 54 \text{ in. or } 4 \text{ ft. } 6 \text{ in.} \end{aligned}$$

Example 45.—Two forces act in a machine as shown graphically in fig. 47 at A and B. Find the magnitude and position of the single force E required to keep them in equilibrium.

For present purposes, downward forces may be considered negative (−), and upward forces positive (+). The *resultant* is the algebraic sum of the forces, i.e.

$$-12 + 8 = -4 \text{ cwt.}$$

The *equilibrant* is therefore + 4 cwt., i.e. a force of 4 cwt. acting in an upward direction.

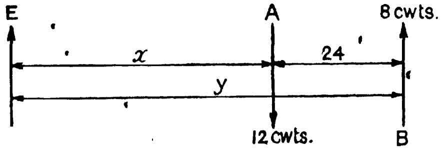

Fig. 47

To find the position of the equilibrant, assume that it acts on the left of A and at a distance x as indicated at E, fig. 47; then the moments about A are:

$$4x = 8 \times 24.$$

$$x = \frac{8 \times 24}{4} = 48 \text{ in.}$$

The position is thus 48 in. to the left of A. This can be verified by taking moments about B as under:

$$4y = 12 \times 24$$

$$y = \frac{12 \times 24}{4} = 72 \text{ in.}$$

The position is thus 72 in. to the left of B, and verifies the assumption made, since the force line shown at E is in the only position which is simultaneously 72 in. from B and 48 in. from A.

Example 46.—A loom crank shaft weighs 80 lb. and is supported in bearings at 5 ft. centres. The pulleys, pinion, &c., weigh 30 lb., and may be taken as acting at a point 12 in. outside one bearing; the hand or fly-wheel weighs 10 lb., and may be assumed to act at a point 6 in. outside the other bearing. If the centre of gravity of the shaft itself is at a point 2 ft. 3 in. inside the pulley bearing, find the pressure on each bearing due to these loads.

Fig. 48 gives a diagrammatic view of the problem where the required pressures on the bearings are equal to the reactions at R and R_1. Particular attention must be paid to the *direction* of the various moments.

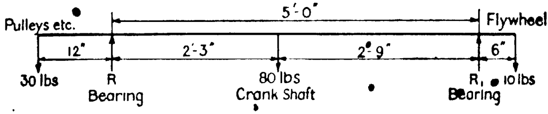

Fig. 48

Apply the principle of moments by taking moments about R_1:

$$5R + (10 \times \tfrac{1}{2}) = (80 \times 2\tfrac{3}{4}) + (30 \times 6).$$
$$5R = 220 + 180 - 5$$
$$= 395.$$
$$\therefore R = \frac{395}{5} = 79 \text{ lb.}$$

Then take moments about R as under:

$$5R_1 + (30 \times 1) = (80 \times 2\tfrac{1}{4}) + (10 \times 5\tfrac{1}{2}).$$
$$5R_1 = 180 + 55 - 30$$
$$= 205.$$
$$\therefore R_1 = \frac{205}{5} = 41 \text{ lb.}$$

Check: $R + R_1 = 79 + 41 = 120$ lb.
and $80 + 30 + 10 = 120$ lb.

COUPLES.—Consider the various pairs of forces indicated in fig. 49, most of the distances of which are introduced solely to illustrate an important principle, and are not to be considered as being of practicable application; other proportionate numbers are, however, correct in practice.

(P). The equilibrant is (10 − 9) = 1 lb. It must act at a point 9 ft. to the left of A.

(Q). The equilibrant is (10 − 9·9) = ·1 lb., acting at a point 99 ft. to the left of A.

(R). The equilibrant is (10 − 9·99) = ·01 lb., acting at a point 999 ft. to the left of A.

(S). The equilibrant, although it may not be so obvious as in the first 3 cases, is (10 − 10) = 0 lb. acting at a point at an *infinitely* great distance to the left of A.

The result in (S) may be deduced from a comparison of those in (P), (Q), and (R). As the two forces become more and more nearly equal, the equilibrant becomes smaller and smaller, but acts at an increasingly greater distance to the left of A. When the forces become equal, the equilibrant becomes infinitely small (in the limit 0, or zero) and the distance becomes infinitely great (in the limit ∞, or infinity).

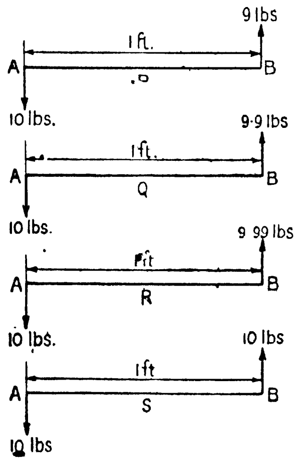

Fig. 49

Two such equal and parallel forces acting along separate lines in opposite directions, as at S, fig. 49, constitute what is known as a *Statical or Mechanical Couple*. The many properties of couples are both interesting and useful, but a full considera-

tion of them cannot appear at this stage. It will be sufficient for the present to indicate a few of their most characteristic properties, and to show one example from a well-known textile mechanism. (1) When two parallel forces, equal in magnitude, but opposite in direction, act on a body, they produce, or tend to produce, rotation. It will be shown that the converse is also true, namely, that before rotation can be produced a couple must be originated. (2) The *moment of a couple* is the product of *one* of the forces and the perpendicular distance between them. This distance is termed the *arm* of the couple. (3) No single force can be applied to balance a couple; the couple can only be balanced by the application of a second couple, equal in magnitude, but opposite in direction.

These three chief properties of couples may be illustrated by reference to fig. 50, which is a skeleton diagram of the loom mechanism shown more fully in fig. 1. The loom crank C is rotated, say, in the direction indicated by the arrow D. The crank C actuates a sword S, and the latter, which carries the slay or lay, is fulcrumed on the rocking shaft Q. The motion of the crank C is conveyed to the sword S by means of a connecting arm A. The turning force in the crank C acts tangentially to the crank circle, i.e. at right angles to the crank, as indicated by the force line F. The turning force F has a moment about the crankshaft centre O, in consequence of which the crank C rotates about O. But every force, as an essential condition of existence, must have a corresponding reaction, equal in magnitude but opposite in direction. In the case of the crank, this reaction can only exist in the crankshaft bearing; it may, therefore, be represented by a force-line R, representing a force equal in magni-

tude to F, but opposite in direction. The force F and the reaction R thus form a couple, the moment of which is measured by the throw of the crank C, and the effect of which, in the present instance, is to cause rotation about the point O, in a counter-clockwise direction.

Again, in the particular position indicated, *part* of the force F, viz. F_1, is transmitted to the connecting arm A, to overcome the resistance R_1 set up by the slay, sword, &c. This resistance R_1 obviously could not exist if there were not a corresponding reaction, equal in magnitude, but opposite in direction, set up simultaneously in the bearing R_2 of the rocking shaft Q. The forces R_1 and R_2 constitute a second couple tending to produce rotation in a clockwise direction.

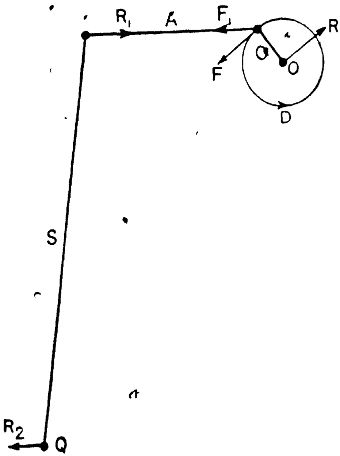

Fig. 50

Each instance illustrates the fact that if a body is to rotate, there must be a couple acting on it. Again, since the complete arrangement at any instant is in equilibrium, both instances serve to demonstrate that one couple is balanced by a second couple, equal in magnitude, but opposite in direction.

Exercises, with answers, on p. 164.

CHAPTER IX

THE PRINCIPLE OF WORK

The Principle of Work is axiomatic in nature, for it simply asserts an obvious truth. Its very simplicity, however, is misleading, since the majority of students, on making acquaintance with it for the first time, do not realize its extreme importance. The principle may be asserted in many different ways; that immediately following will perhaps be most readily understood by elementary students.

The Principle of Work asserts that the total work put into a machine is invariably equal to the work got out of the machine plus the work done in overcoming the internal resistances of the machine. The work got out of the machine may conveniently be called the *Useful Work*; that is to say, it is the work for which the machine is actually designed. The work done in overcoming the internal resistances of the machine is often termed *Lost Work*; it is work done incidentally to the chief purpose of the machine, not because it is *necessary* for the ordinary purposes of the machine, but because it is unavoidable.

The Principle of Work may therefore be concisely summed up as under:—

Work put in = Useful Work + Lost Work.

WORK PUT IN. — When a machine, such as a spinning frame of any kind, is employed in the doing of work, force must first of all be conveyed to the machine. This driving force may be transmitted by a belt to the fast pulley of the mule, and by means of gears, shafts, &c., it is transmitted to all the component parts of the machine, causing them to

move in their appointed fashion, and to do the work of drawing, twisting, and coiling the finished material, for the spinning of which the machine is intended. In short, the work done by the force in the moving belt is given to the machine at one part, and this force is constrained and modified by the different mechanisms of the machine in order to do various kinds of work at other parts.

The magnitude of this force, multiplied by the distance through which it acts, is equal to the work put into the machine. If the force varies, or if the distance varies, in a complete cycle of operations, then, as previously shown, it is the *average* force or the *average* distance upon which calculations should be based.

Lost Work.—The function of a spinning-mule is to draw down the rove into a size corresponding to the count of yarn required, to twist this fine modified sliver into a yarn, and to coil the yarn into the well-known cop shape on the spindle. Now, even if no material were in the machine, i.e. if no yarn were being spun, and the mule were set in motion, a comparatively large amount of force would be required to drive it. This force would be used up in overcoming the inertia of the moving parts, in reversing the motions of the carriage, in overcoming the friction of all parts running in bearings, such as the spindles in their bolsters, &c. The chief point to notice is that this work is not considered as *useful work*, even though it is absolutely necessary. This work is termed the *lost work*, and it varies in amount according to the condition of the machine, the attention it receives, the method of driving it, &c.

In the design of textile machinery, or indeed of any kind of machine, it is most important that this

lost work should be as small as possible. Many devices are in existence for minimizing the amount of lost work, but it has not been found possible to obviate entirely the lost work.

USEFUL WORK.—The difference between the work put in and the lost work is the useful work. It is the actual work done by the machine in fulfilling its designed function. In the case of the spinning-mule, it is the actual amount of work done in drawing the rove down to the required size, in twisting this into yarn, and in coiling up the yarn on the spindle in the form of a cop.

The higher the proportion of useful work with regard to work put in, the more perfect is the machine. A perfect machine, i.e. one in which there is no lost work, is taken as having an efficiency of 100 per cent. Since there must always be some loss of work, actual efficiencies are all below 100 per cent.

EFFICIENCY.—The efficiency of a machine is thus the *ratio* of the useful work to the work put in. Thus, if 100 ft.-lb. of work are put into a machine, and 80 ft.-lb. of useful work are obtained from the machine, its efficiency is:

$$\text{Efficiency} = \frac{\text{Useful work}}{\text{Work put in}} = \frac{80 \text{ ft.-lb.}}{100 \text{ ft.-lb.}} = \frac{4}{5}.$$

or 80 per cent.

The application of the Principle of Work to the solution of practical problems may best be shown by taking as an example another of the simple machines or elementary mechanisms, viz. the wheel and simple axle.

WHEEL AND SIMPLE AXLE.—The wheel and axle is a very old contrivance for lifting weights; it has

been used for hundreds of years to lift water from wells; it is still in use in some textile factories to lift bags of farina, flour, &c., used in the starching or slashing of warps, into the storehouse above the starch-mixing apparatus.

A diagrammatic representation of one modern form of wheel and simple axle is shown in fig. 51. The device consists of a chain, barrel, or axle A,

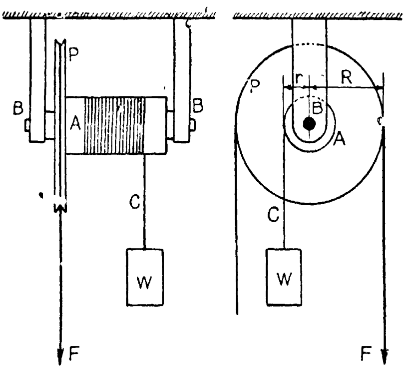

Fig. 51

running in suitable bearings B, a chain C fixed at one end to the barrel A and coiled round it, while the other end of the chain C hangs free and usually terminates in a hook, a grooved pulley P firmly fixed to the barrel, and a rope, usually endless in form, for operating the pulley P. If a force F is applied to the rope, as suggested in the side elevation, the whole mechanism will rotate and coil the chain C on to the barrel A; the device may thus be used for hoisting all kinds of goods to elevated positions.

If it is assumed for the moment that there is no

lost work, i.e. that the device has an efficiency of 100 per cent, then the Principle of Work asserts that the work put in is exactly equivalent to the work got out. The action of this simple machine, and of all other machines, is best studied by assuming that one complete cycle of operations has taken place. In the present instance, one complete cycle would be represented by one complete revolution of the wheel P.

During one revolution of the wheel, the work put in is evidently equal to the magnitude of F multiplied by the distance through which it moves. Since the *effective* movement is represented by the circumference of the wheel, it follows that:

$$\text{Work put in} = F \times 2\pi R \text{ or } F \times \pi D,$$

where R is the effective radius of the wheel, and D its diameter.

Now during this revolution of the wheel P, the barrel or axle A will also have completed one revolution, and a length of chain C, equal to the *effective* circumference of the barrel A, will have been coiled on it, thus raising the load W a corresponding distance. Hence:

$$\text{Work got out} = W \times 2\pi r,$$

where r is the effective radius of the barrel A.

If we keep in mind the above-mentioned assumption, and the Principle of Work as there stated, we shall see that:

$$\begin{aligned} \text{Work put in} &= \text{Work got out}, \\ \text{i.e. } F \times 2\pi R &= W \times 2\pi r, \\ \text{whence } FR &= Wr, \\ \text{or } \frac{F}{W} &= \frac{r}{R}. \end{aligned}$$

A further examination of fig. 51 will show that the problem may be regarded as one of moments applied to a two-armed lever; one arm is R, the effective radius of the wheel P, and the other arm is r, the effective radius of the barrel A. To the arm R is applied a force F, and to the arm r a load or resistance W. At the instant the wheel begins to move, all forces acting on the mechanism are in equilibrium; hence, by the Principle of Moments we have:

$$F \times R = W \times r,$$
$$\text{i.e.} \qquad FR = Wr,$$

so that the Principle of Work and the Principle of Moments are in complete accord with one another.

A wheel or pulley may indeed be regarded as a *continuous* lever—providing means whereby the principle of lever action may be continued for any desired length of time.

Example 47.—A wheel and axle is required so that the force applied at the circumference of the wheel in moving through a distance of 12 ft. shall raise a load of 4 cwt. through a distance of 2 ft. If the effective diameter of the barrel is 8 in., find the force applied in pounds and the diameter of the wheel. Friction, &c., is neglected.

By the Principle of Work:

$$\text{Work got out} = \text{Work put in,}$$
$$\text{whence} \quad 4 \text{ cwt.} \times 2 \text{ ft.} = F \text{ lb.} \times 12 \text{ ft.}$$
$$12\,F = 4 \times 112 \text{ lb.} \times 2 \text{ ft.}$$
$$\therefore F = \frac{4 \times 112 \times 2}{12}$$
$$= 74\tfrac{2}{3} \text{ lb.}$$

Now take moments about the arbor of the wheel and apply the Principle of Moments, thus:

$$74\tfrac{2}{3} \text{ lb.} \times R = 448 \text{ lb.} \times 4 \text{ in. } (\textit{radius} \text{ of barrel}).$$

$$R = \frac{448 \times 4}{74\frac{2}{3}}$$

$$= \frac{448 \times 4 \times 3}{224} = 24 \text{ in.},$$

whence the diameter of the wheel

$$= 24 \times 2 = 48 \text{ in.} = 4 \text{ ft.}$$

Example 48.—The top rollers in a plain tappet or wyper loom are $1\frac{1}{4}$ in. and $1\frac{1}{2}$ in. diameter respectively, as shown in fig. 52; the former is attached to the front leaf, and the latter to the back leaf. If it requires a pull of 10 lb. to raise the back leaf from its low position to its high position, find the downward pull on A while B is being raised.

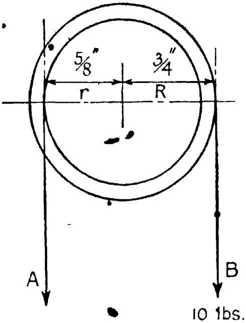

Fig. 52

The top roller is evidently only a special form of wheel and axle, so that the Principle of Work (or the Principle of Moments) may be applied as in the previous example.

By the Principle of Work:

$$Fr = WR$$

$$F = \frac{WR}{r}$$

$$= \frac{10 \times \frac{3}{4}}{\frac{5}{8}}$$

$$= \frac{10 \times 3 \times 8}{4 \times 5} = 12 \text{ lb.}$$

Example 49.—Find the average horse-power exerted by an electric motor in raising a hoist and case of textile machinery, weighing 3 tons, a distance of 60 ft., at a uniform speed in $1\frac{1}{2}$ minutes, supposing that 25 per cent of the total work done is lost in overcoming friction, &c.:

$$\text{Useful H.P.} = \frac{R \times S}{33{,}000}$$
$$= \frac{3 \text{ tons} \times 60 \text{ ft.}}{33{,}000 \times 1\frac{1}{2} \text{ min.}}$$
$$= \frac{3 \times 2240 \times 60 \times 2}{33{,}000 \times 3}$$
$$\doteqdot 8{\cdot}15 \text{ H.P.}$$

Total horse-power = useful horse-power + lost horse-power, i.e. 100 per cent = 75 per cent + 25 per cent; so that useful horse-power = 75 per cent, and the total horse-power exerted by the motor is:

$$\frac{8{\cdot}15 \times 100 \text{ per cent}}{75 \text{ per cent}} \doteqdot 10{\cdot}87 \text{ H.P.}$$

Exercises, with answers, on p. 164.

CHAPTER X

PULLEYS: SIMPLE AND COMPOUND. MECHANICAL ADVANTAGE AND VELOCITY RATIO

Suppose that the distance between the store-room floor of a factory and the roadway beneath it is 20 ft., then all goods delivered to the factory have to be raised 20 ft. Consider the following three methods of raising, say, a case of buffalo-hide pickers weighing 60 lb.

1. A man is employed to do the work by carrying the case on his shoulder up a stairway. His own weight is 10 st. 10 lb. = 150 lb. Now:

Work done in raising the pickers	= 60 lb. × 20 ft. = 1200 ft.-lb.
Work done in raising himself	= 150 lb. × 20 ft. = 3000 ft.-lb.
Total work done	= 210 lb. × 20 ft. = 4200 ft.-lb.

The useful work done is thus 1200 ft.-lb. out of a total of 4200 ft.-lb., i.e. the mechanical efficiency of the method is

$$\frac{\text{Useful work}}{\text{Total work}} = \frac{1200}{4200} = \frac{2}{7},$$

$$\text{and } \frac{2}{7} \times 100 \doteqdot 28{\cdot}57 \text{ per cent.}$$

Notice that	Total work =	100 per cent.
	Useful work =	28·57 ,,
whence,	Lost work =	71·43 ,,

2. A smooth iron hook is fixed into the wall above the doorway of the store-room, and a strong rope passed over the hook. The man attaches one end of the rope to the case and pulls on the other end to raise the case. It is found that a force of 90 lb. has to be exerted. In this case:

Work done in raising the pickers	= 60 lb. × 20 ft. = 1200 ft.-lb.
Total work done	= 90 lb. × 20 ft. = 1800 ft.-lb.
Work done in overcoming friction, in bending the rope, &c.	= 30 lb. × 20 ft. = 600 ft.-lb

The mechanical efficiency of this method is therefore:

$$\frac{1200 \text{ ft.-lb. useful work}}{1800 \text{ ft.-lb. total work}} = \frac{2}{3}$$

$$\text{and } \frac{2}{3} \times 100 \doteqdot 66{\cdot}67 \text{ per cent.}$$

The mechanical efficiency has thus been increased from 28·57 per cent to 66·67 per cent, the increase being due mainly to the fact that the man need not raise his own weight while raising the case.

3. Suppose that the hook is replaced by a cast-iron grooved pulley of suitable diameter, running freely on a central stud carried in a bracket fixed to the wall. One end of a rope passed over the pulley is attached to the case, and the other end is pulled downwards in order to raise the case. Due to the easy running of the pulley on its stud, and to the bending of the rope round a much larger circumference, the lost work is correspondingly reduced, and the force required is found to be, say, 70 lb. Here:

Total work done	= 70 lb. × 20 ft. =	1400 ft.-lb.
Work done in raising the case	= 60 lb. × 20 ft. =	1200 ft.-lb.
Work done in overcoming friction, &c.	= 10 lb. × 20 ft. =	200 ft.-lb.

The mechanical efficiency is therefore

$$\frac{1200 \text{ ft.-lb. useful work}}{1400 \text{ ft.-lb. total work}} = \frac{6}{7}$$

$$\text{and } \frac{6}{7} \times 100 \doteqdot 85{\cdot}71 \text{ per cent.}$$

Although the figures used above are in the main assumed, the results are, nevertheless, approximately correct, and serve to show the usefulness of another type of mechanical element, viz. the Simple Pulley.

Simple Pulley.—The simple pulley, as described in (3) above, is useful for two reasons: (*a*) it reduces the percentage of lost work, i.e. it has a high efficiency compared with other methods of raising loads; and (*b*) it enables the direction of the force to be easily changed. The force applied is invariably greater than the load, and, as other mechanisms enable a load to be raised with a small force, it is mainly employed to take advantage of the property of changing the direction of a force. It is for this latter purpose that it is often used *in conjunction with* another type of mechanism, e.g. a hand or steam winch, a ship's derrick, positive centre-shedding dobbies, the spindles of a spinning frame, &c.

A pulley may be regarded as a special form of the wheel and simple axle where the radii of the wheel and axle are equal. It may also be regarded as a two-armed lever with equal arms. It may still further be considered as a continuous lever, as exemplified in Chapter IX.

The Snatch-block.—What is termed a snatch-block is essentially a simple pulley carried in a frame and from which a hook may be hung. It is used in many lifting appliances, and is often arranged so that the hook may swivel on its axis. When the snatch-block is employed alone, its usefulness is limited, but when arranged as indicated in fig. 53, the state of affairs is much different.

In fig. 53 a rope R is carried over a simple pulley P running in bearings attached to the beam B; from this pulley the rope passes under a snatch-block S, and is then fixed to an eyebolt E, also attached to the beam B. Both pulleys are free to rotate on their respective pins; in addition the snatch-block S can be raised or lowered. A force F may therefore be used in order

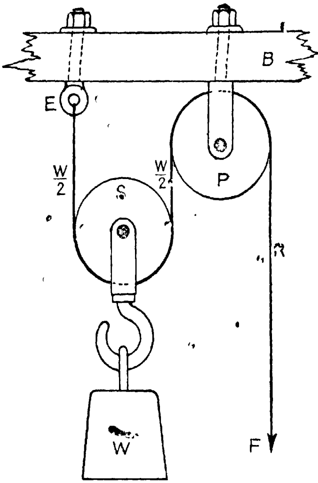

Fig. 53

to raise a load W.

Suppose that a load of W lb. is hung on the hook as indicated; the load is supported by a corresponding force F. If friction, &c., be neglected, i.e. if the mechanism be regarded as perfect, the tension in the rope must be the same in all its parts.

Now it is evident that the load W is supported by two parts of the rope. The load on each part must therefore be half of W, i.e. $\frac{W}{2}$ lb. as shown, so that $\frac{W}{2}$ lb. is the stress existing in *all* parts of the rope; hence the force F must also equal $\frac{W}{2}$ lb. Again, if the force F move downwards through a distance, say, of 2 ft., the load will evidently rise 1 ft., since the 2 ft. of rope has been withdrawn from two parts, and therefore 1 ft. from each part constitute the 2 ft. pulled down at F.

MECHANICAL ADVANTAGE.—If friction, &c., be neglected, then by the Principle of Work:

$$\text{Work put in} = \text{Work got out.}$$
$$F \times 2 \text{ ft.} = W \times 1 \text{ ft.}$$
$$2F = W.$$
$$\therefore F = \frac{W}{2}.$$

That is to say, a force F of 30 lb. would be sufficient to raise a load or weight W of 60 lb.; the force used is thus exactly *half* the load lifted, and such an arrangement is therefore an advantage in raising heavy loads. Indeed, the ratio of W to F is termed in Mechanics the *advantage*, or the *Mechanical Advantage* of the machine. In the above case it is seen that

$$F = \frac{W}{2},$$

whence $\frac{W}{F} = 2 =$ Mechanical Advantage.

Since no account has been taken of the lost work due to frictional resistance, &c., this may be better defined as the *Theoretical Mechanical Advantage*, i.e. the advantage which would obtain if the machine were perfectly efficient.

N.B.—

T.M.A. may be used for Theoretical Mechanical Advantage.

A.M.A. may be used for Actual Mechanical Advantage.

VELOCITY RATIO.—In the case of the snatch-block in fig. 53, it was shown that, when F moved through 2 ft., W moved through 1 ft. The ratio of the movement of W to that of F is constant in any particular machine, since it is determined solely by the dimensions, location, and design of particular parts. This ratio is termed the *Velocity Ratio*; in every case it is represented by:

$$\frac{\text{Rate of movement of W}}{\text{Rate of movement of F}}.$$

In all cases the Velocity Ratio is the reciprocal of the

T.M.A. That is to say, if the T.M.A. is

$$\text{the V.R is } \frac{1}{x}.$$

(V.R. may be used as an abbreviation of Velocity Ratio.)

Rope Tackle.—In lifting appliances of the kind exhibited in fig. 53, the mechanical advantage can be increased to as large a figure as is desired or practicable by compounding, or using in combination, any suitable number of pulleys, but only one is used to any extent in practice, viz. the ordinary form of block and tackle worked by a rope.

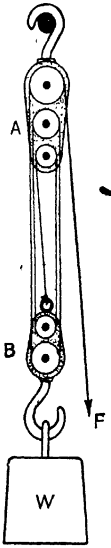

Fig. 54

Fig. 54 is a diagrammatic representation of a "3 and 2" rope tackle. It consists of two sheaves, an upper fixed sheave A carrying 3 pulleys on a stud, and a lower movable sheave B carrying 2 pulleys. In fig. 54 the pulleys are shown of different sizes, and on different centres, in order to show distinctly the path of the rope; it will be understood, however, that all the pulleys in each sheave are of the same diameter and run loosely on the same stud or pin. The rope is attached at one end to a ring in the lower block or sheave, and is then arranged over and under the pulleys in the manner indicated. A downward force applied at F is capable of raising loads attached at W.

The load W is thus supported by 5 parts or stretches of the rope, i.e. there are 5 stretches of rope between the two sheaves A and B, so that the tension in each

part or stretch of the rope is $\frac{W}{5}$; from this it follows that F is equal to $\frac{W}{5}$ if friction, &c., be neglected.

The T.M.A. of the machine is the ratio $\frac{W}{F}$; hence, if

$$F = \frac{W}{5},$$

$$\text{then} \qquad \frac{W}{F} = 5 = \text{the T.M.A.}$$

But 5 is the number of stretches of rope between the two sheaves A and B, so that in the case of a rope tackle

the T.M.A. = the number of stretches or parts of the rope supporting the load.

The V.R. is the ratio of the distance moved by W in a given time compared with the distance moved by F in the same time. If F is drawn downwards through 10 ft., then W will be raised 2 ft., since the 10 ft. pulled down was, immediately before the action, spread over the 5 parts of rope between the sheaves; hence $\frac{10 \text{ ft.}}{5} = 2$ ft.

The V.R. is thus $\frac{2 \text{ ft.}}{10 \text{ ft.}} = \frac{1}{5}$, and in the case of a rope tackle this is the same as

$$\frac{1}{\text{number of parts of rope supporting load}}.$$

Example 50.—In an ordinary block and tackle, such as that shown in fig. 54, a pull of 50 lb. is required in order to raise a shaft weighing 165 lb. Find the following: (1) the T.M.A.; (2) the V.R.; (3) the actual mechanical advantage (A.M.A.) of this load;

(4) the efficiency at this load; and (5) the percentage efficiency at this load.

1. T.M.A. = Number of rope parts supporting load

$$= 3 + 2 = 5.$$

2. V.R. $= \dfrac{1}{\text{T.M.A.}} = \dfrac{1}{5}.$

3. A.M.A. $= \dfrac{\text{Magnitude of W}}{\text{Magnitude of F}}$

$$= \frac{165 \text{ lb.}}{50 \text{ lb.}} = 3 \cdot 3.$$

4. Efficiency $= \dfrac{\text{A.M.A.}}{\text{T.M.A.}}$

$$= \frac{3 \cdot 3}{5} = \cdot 66,$$

or $\dfrac{\text{actual load}}{\text{theoretical load}}$

$$= \frac{165}{5 \times 50} = \cdot 66.$$

5. Percentage Efficiency $= \dfrac{\text{A.M.A.}}{\text{T.M.A.}} \times 100$

$$= \frac{3 \cdot 3}{5} \times 100$$

$$= 66 \text{ per cent.}$$

Example 51.—A rope tackle consists of 2 blocks or sheaves; the upper or fixed one weighs 16 lb. and contains 2 pulleys, while the lower or movable one weighs 13 lb., and carries one single movable pulley. The parts of the rope are vertical, and the standing end is fixed to the lower block. What pull on the rope will support a load of 200 lb., and what will then be the pressure on the point of support? The efficiency of the apparatus at this load is 70 per cent.

$$\text{T.M.A.} = \text{number of parts of rope supporting load, or number of pulleys,}$$
$$= 3.$$

$$\text{Now T.M.A.} = \frac{W}{F}.$$

$$\therefore \frac{W}{F} = 3,$$

$$\text{whence, } F = \frac{W}{3}.$$

Note that, in lifting the load of 200 lb., the lower block, which weighs 13 lb., must also be lifted. Therefore

$$F = \frac{200 \text{ lb.} + 13 \text{ lb.}}{3}$$
$$= \frac{213}{3} = 71 \text{ lb.}$$

But 71 lb. is correct only if the machine were perfectly efficient; the machine has an efficiency of only 70 per cent, therefore this force must be increased in the ratio of 100 to 70.

$$\text{Actual F} = 71 \times \frac{100}{70}$$
$$= \frac{7100}{70} \doteqdot 101{\cdot}43 \text{ lb.}$$

$$\text{Pressure on support} = \text{Weight of 2 blocks + load + force.}$$
$$= (16 \text{ lb.} + 13 \text{ lb.}) + 200 \text{ lb.} + 101{\cdot}43 \text{ lb.}$$
$$= 330{\cdot}43 \text{ lb.}$$

The efficiency of rope tackles varies from 40 to 70 per cent, and, as in all other machines, increases with the load. Indeed, with the loads for which such a lifting appliance is almost invariably used, the

efficiency exceeds 50 per cent. When the efficiency does exceed 50 per cent, a device of this kind becomes reversible, i.e. when the force F is taken away, the load W will descend of its own accord. This fact should be kept in mind to compare with particulars of a *chain tackle* to be given in a succeeding chapter.

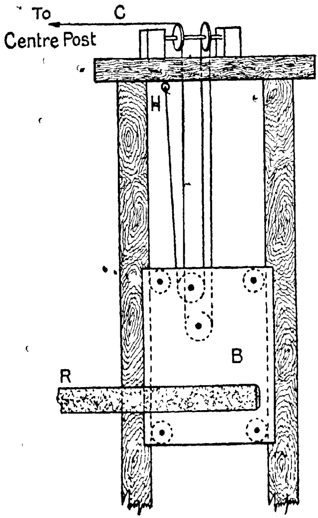

Fig. 55

An apparatus, somewhat different in function to the above, but identical in principle, is used for the heck-box of an ordinary vertical warping mill, the essential parts of which are shown in fig. 55. A cord C is wound round the centre-post of the mill, and passes over 4 pulleys as shown, while the standing end is fixed to a ring H. The pins, upon which the two lower pulleys rotate, are carried by the heck-box B itself. A rod or bar R on the heck-box B supports the two guide and leasing reeds required in the working of the mill, or rather in the manipulation of the warp threads. As the centre-post revolves, it gives off, or takes on, a definite length of cord every revolution, and lowers, or raises, the heck-box B a distance corresponding to the length, but modified by the

"trim". Fig. 55 shows what is termed a 4-cord trim. Each inch of cord given off or taken on by the centre-post is spread over these 4 cords, and results in the heck-box falling or rising $\frac{1}{4}$ in. The purpose of the heck-box is to guide the threads on to the mill in such a way that the combination of a vertical movement with a horizontal movement results in an oblique disposition of the yarn on the mill, while the number of cords determines the pitch of the coils of yarn on the mill, or the distance between successive rounds of yarn.

Example 52.—In a warping mill the effective diameter of the centre-post is 3 in., and the heck-box B is suspended by 4 cords as in fig. 55, i.e. a 4-cord trim. Calculate the distance between successive rounds of yarn.

In one revolution of the mill the centre-post will give off, or take on, 3 in. × 3·1416 = 9·4248 in. This 9·4248 in. is spread over the 4 cords, so that

$$\frac{9{\cdot}4248 \text{ in.}}{4 \text{ cords}} = 2{\cdot}3562 \text{ in.}$$

will be the vertical pitch of the rounds of yarn.

If the working height of the mill is 6 ft., then the maximum number of rounds is

$$\frac{6 \text{ ft.} \times 12 \text{ in. per foot}}{2{\cdot}3562 \text{ in. pitch}} \doteq 30{\cdot}56 \text{ rounds.}$$

And, if the mill is 10 yd. in circumference, the longest chain which can be made with this 4-cord trim is approximately

$$30{\cdot}56 \times 10 \text{ yd.} = 305{\cdot}6 \text{ yd.}$$

Exercises, with answers, on p. 165.

CHAPTER XI

WHEEL AND COMPOUND AXLE
DIFFERENTIAL PULLEY BLOCK

WHEEL AND COMPOUND AXLE.—This device, sometimes termed the Chinese Windlass, may be regarded as a development of the Wheel and Simple Axle. The mechanism itself is extremely ancient, and is now seldom, if ever, used in Western countries. But, because a new principle is introduced, and because it possesses a modern prototype in the Weston Differential Pulley Block, it is well worth the attention of the student.

The essential parts of a wheel and compound axle are shown in fig. 56. It consists of an arbor A running in suitable bearings B. On the arbor is mounted a barrel having two diameters, a small one at S and a large one at L. At one end of the arbor A is fixed a grooved wheel G, and a rope runs in this groove to put the whole mechanism in motion. When this rope is pulled in the direction indicated at F, a second rope is coiled on the large barrel L and unwound from the small barrel S. Since for each revolution more rope is wound on L than is unwound from S, the load W, fixed to the hook of a single movable pulley P, is gradually raised.

Note.—*In fig 56, the single pulley P in the side elevation is purposely out of projection (90° round) in order to show more clearly the path followed by the rope to the two diameters of the barrel.*

It will be perceived that the windlass has a *differential* action; the net amount of rope coiled up, and consequently the distance through which

the load is raised in one revolution of the barrel, is dependent entirely on the *difference* in the diameters of the two parts L and S of the compound axle.

RELATION OF W TO F.—The relation of W to F

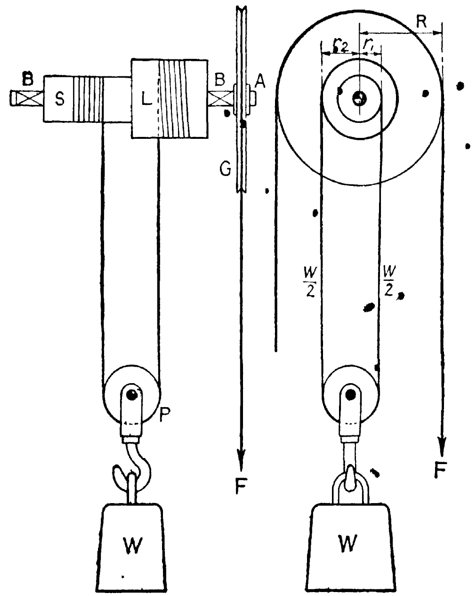

Fig. 56

may be established by applying either the Principle of Moments or the Principle of Work. Let F be the force required to start the load W; let R be the radius of the wheel G, r_1 the radius of the small axle S, and r_2 the radius of the large axle L. Note that the load W is supported by two parts of the

winding rope, so that the tension in the winding rope at all parts is $\frac{W}{2}$.

Now, applying the Principle of Moments, and assuming that the apparatus is perfectly efficient, the Sum of Clockwise Moments = Sum of Counter-clockwise Moments, i.e.

$$(F \times R) + \left(\frac{W}{2} \times r_1\right) = \frac{W}{2} \times r_2.$$

$$FR = \frac{W}{2}r_2 - \frac{W}{2}r_1$$

$$= \frac{W}{2}(r_2 - r_1),$$

$$\text{whence, } F = \frac{W(r_2 - r_1)}{2R}.$$

Example 53.—In a wheel and compound axle, such as that illustrated in fig. 56, the diameters of the two portions L and S of the axle are 3 in. and 2 in. respectively, and the wheel G, which rotates the axle, has a radius of 12 in. If a force of 20 lb. be applied to a point on the circumference of the wheel G, find what weight will be raised. Friction, &c., is neglected.

$$F = \frac{W(r_2 - r_1)}{2R}.$$

$$W(r_2 - r_1) = F2R.$$

$$\therefore W = \frac{2FR}{r_2 - r_1}$$

$$= \frac{2 \times 20 \times 12}{1\frac{1}{2}\text{ in.} - 1\text{ in.}}$$

$$= \frac{2 \times 20 \times 12}{\frac{1}{2}}$$

$$= 960 \text{ lb.}$$

Again, the Principle of Work may be applied. Neglect friction and suppose the rope to be perfectly flexible—in other words, assume a perfect machine (100 per cent efficiency), and cause the wheel G to make one complete revolution. Then, by the Principle of Work:

$$\begin{aligned} \text{Work put in} &= \text{Work got out,} \\ \text{i.e. } F \times \text{its distance} &= W \times \text{its distance.} \end{aligned}$$

F × circumference of wheel G = W × $\frac{1}{2}$ difference of axle circumferences L and S. ($\frac{1}{2}$ because of single movable pulley.)

$$\begin{aligned} F \times 2\pi R &= W \times \tfrac{1}{2}(2\pi r_2 - 2\pi r_1). \\ 2\pi FR &= \frac{W}{2} \times 2\pi(r_2 - r_1). \\ FR &= \frac{W}{2}(r_2 - r_1), \\ \text{whence, } F &= \frac{W(r_2 - r_1)}{2R}. \end{aligned}$$

This result is absolutely identical with the value of F obtained by using the Principle of Moments, so that both principles are again in complete agreement.

Example 54.—In a compound wheel and axle, the diameter of the larger axle L is 6 in.; of the smaller axle S, 3 in.; and of the wheel G, 36 in.; find the T.M.A. and the V.R. of the machine.

By the Principle of Work

$$F = \frac{W\,(r_2 - r_1)}{2\,R},$$

and, as previously shown,

$$\text{T.M.A.} = \frac{W}{F}.$$

$$\text{Now, } W(r_2 - r_1) = 2FR,$$

$$W = \frac{2FR}{r_2 - r_1},$$

$$\text{and } \frac{W}{F} = \frac{2R}{r_2 - r_1} \quad \text{.... (see (*) below.)}$$

$$= \frac{2 \times 18}{3 - 1\frac{1}{2}}$$

$$= \frac{36}{1\frac{1}{2}}$$

$$= 24 = \text{T.M.A.}$$

$$\text{Velocity Ratio} = \frac{1}{\text{T.M.A.}} = \frac{1}{24}.$$

The results imply that (1) a force of 1 lb. will raise a load of 24 lb., and (2) while the load moves 1 ft. the force will move through 24 ft. In practice the first result will be modified by the actual efficiency of the machine at any particular load; the second result will always obtain, since, as shown indirectly at (*) above, it is dependent solely on the *dimensions* of the machine.

Although this type of lifting machine can give a big mechanical advantage, and a fairly high efficiency, its use is precluded in practice by the large length of rope required to lift loads through comparatively short distances. It is now completely replaced by the Weston Differential Pulley Block.

THE WESTON DIFFERENTIAL PULLEY BLOCK.—The Weston Block, a diagrammatic view of which appears in fig. 57, consists essentially of three main parts:

(1) An upper block B;
(2) A lower block P; and
(3) An endless chain C.

The upper block carries two grooved pulleys of

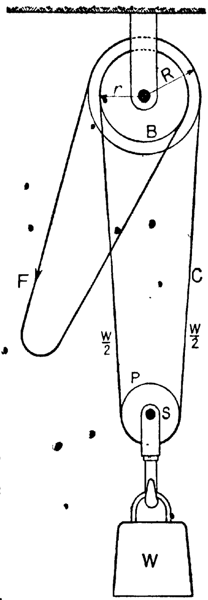

Fig. 57

different diameters compounded, i.e. fixed together, and are provided with depressions into which drop the links of the chain; all slipping is thus prevented. The lower block is a single movable pulley or snatch block; the groove is smooth and serves to guide the chain. A hook of the usual form depends from the stud S and carries the load W. The chain is of mild steel, of uniform pitch and size of link. The complete apparatus may be regarded as a Chinese windlass without a wheel.

RELATION OF W TO F.—Refer to fig. 57 and apply the Principle of Work; thus:

$$\text{Work put in} = \text{Work got out.}$$

$$F \times \text{its distance} = W \times \left\{\text{its distance.}\right.$$

$$F \times \left.\begin{matrix}\text{circumference of} \\ \text{large pulley}\end{matrix}\right\} = \frac{W}{2} \times \left\{\begin{matrix}\text{difference between} \\ \text{the circumference} \\ \text{of both pulleys;}\end{matrix}\right.$$

$$\text{hence, } F \times 2\pi R = \frac{W}{2} \times (2\pi R - 2\pi r),$$

$$2\pi FR = \frac{W}{2} 2\pi (R - r),$$

$$FR = \frac{W}{2} (R - r),$$

$$F = \frac{W(R - r)}{2R}.$$

Example 55.—In an experiment with a Weston

Block, having in the upper block one pulley of 4 in. effective radius (pulley centre to chain centre), and the other pulley of $3\frac{1}{2}$ in. effective radius, a total load of 120 lb. (deadweight + lower block + chain), can just be lifted with a pull of 20 lb. Find: (1) the T.M.A.; (2) the V.R.; (3) the A.M.A.; and (4) the percentage efficiency at this load.

(1) $$\text{T.M.A.} = \frac{W}{F},$$

and by the Principle of Work:

$$F = \frac{W(R - r)}{2R},$$

$$\text{whence } W(R - r) = 2RF,$$

$$\text{and } \frac{W}{F} = \frac{2R}{(R - r)}$$

$$= \frac{2 \times 4}{4 - 3\frac{1}{2}}$$

$$= \frac{8}{\frac{1}{2}} = 16 = \text{T.M.A.}$$

(2) $$\text{V.R.} = \frac{1}{\text{T.M.A.}} = \frac{1}{16},$$

i.e. the load moves 1 ft. for each 16 ft. through which the force F moves.

(3) $$\text{A.M.A.} = \frac{\text{actual load lifted}}{\text{actual force used}}$$

$$= \frac{120 \text{ lb.}}{20 \text{ lb.}} = 6 = \text{A.M.A.}$$

(4) $$\left.\begin{matrix}\text{Percentage}\\ \text{Efficiency}\end{matrix}\right\} = \frac{\text{Work got out}}{\text{Work put in}} \times 100$$

$$= \frac{120 \text{ lb.} \times 1 \text{ ft.}}{20 \text{ lb.} \times 16 \text{ ft.}} \times 100$$

$$= \frac{120 \text{ ft.-lb.}}{320 \text{ ft.-lb.}} \times 100$$

$$= 37 \cdot 50 \text{ per cent efficiency.}$$

Alternatively:

$$\text{Efficiency} = \frac{\text{Actual M.A.}}{\text{T.M.A.}} \times 100$$

$$= \frac{6}{16} \times 100 = 37{\cdot}50 \text{ per cent.}$$

In general, such lifting appliances have low efficiencies, but this is not wholly a disadvantage, since with an efficiency below 50 per cent the machine becomes irreversible. In the present instance, a load which has been lifted will remain at any position after the force has been withdrawn. This is due to the fact that the chain cannot slip round the pulleys in the upper block, and to the large amount of friction in the upper block, which is so great that more than 50 per cent of the work put in is used up in overcoming it. The friction is due mainly to the pressure created by the load. In the example just worked out (No. 55), the total load on the upper block is 120 lb. (W), and 20 lb. (F), equal to a total of 140 lb. Remove F, and there is still 120 lb. pressure on the block, that is, there is still $\frac{120}{140}$ or $\frac{6}{7}$ of the total pressure on the block. To obtain 120 ft.-lb. of work, 320 ft.-lb. of work were put in, i.e. 200 ft.-lb. were lost. To overcome the friction in lowering the load, at least $\frac{6}{7}$ of 200 lb.; or, say, 172 ft.-lb. of work must be put in. The load itself can account for 120 ft.-lb., so that $(172 - 120) =$ 52 ft.-lb. of work must be put in before the load can be lowered.

If the student has access to an ordinarily equipped mechanical laboratory, he may learn much by conducting an experiment on a Weston block, or other lifting tackle in a similar way to that described below. The figures and curves given have been obtained from an actual experiment carried out on the lines indicated.

Example 56.—To determine experimentally the Velocity Ratio, the Mechanical Advantage, the Efficiency, and the Mechanical Law of a Weston Pulley Block.

APPARATUS.—The apparatus consists mainly of a small Weston pulley block arranged as in fig. 58 opposite. The block B is hung from a hook H. A spring balance S is attached to the pulling side of the chain, and to a wooden lever L fulcrumed at P in a stand T; the balance serves to register the pull required to raise the load W.

VELOCITY RATIO.—Allow the weight W just to touch the floor. Pull in a definite length of chain, and note the distance through which W has been raised. Repeat this operation at least three times as a check on results, and calculate the V.R. Tabulate the results as under:—

TABLE IV

Test No.	Distance travelled by F.	Distance travelled by W.	V.R.
1	39·37 in.	2·7 in.	$\frac{1}{14·58}$
2	24·0 in.	1·65 in.	$\frac{1}{14·55}$
3	48·0 in.	3·3 in.	$\frac{1}{14·55}$
	Mean V.R. or Mean Velocity Ratio	=	$\frac{1}{14·56}$

ACTUAL MECHANICAL ADVANTAGE. — Apply say five different loads W. In each case move lever L,

fig. 58, slowly downwards through a small distance, sufficient to raise the weight W from the floor. Note the corresponding reading on the spring balance S. Tabulate the results as follows:

TABLE V

No.	W.	F.
	lb.	lb.
1	14	$5\frac{3}{4}$
2	28	$8\frac{3}{4}$
3	42	$11\frac{1}{4}$
4	56	$12\frac{3}{4}$
5	84	$16\frac{3}{4}$

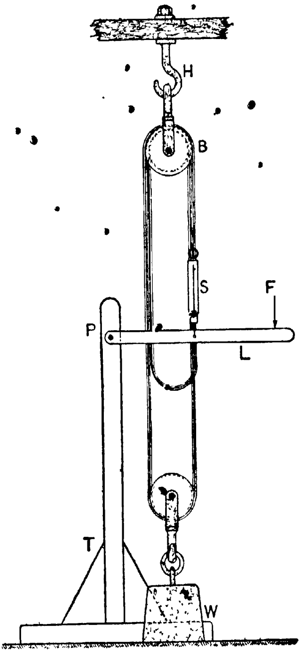

Fig. 58

As the spring balance reading is only correct to $\frac{1}{4}$ lb., it is quite possible that the values thus found may each differ by a small amount from the *real* value; e.g. the $16\frac{3}{4}$ lb. required to raise 84 lb. may actually be 16·7 lb. These values may be checked by plotting an effort curve on squared paper, finding the straight line about which the points representing the values are most evenly distributed, and correcting the values of F from this straight-line graph. This is done in fig. 59, where the small crosses indicate the experimental values in Table V, and the line F is the effort graph.

Raising and Lowering Effort—F and f; 1 in. to 5 lb.
Mechanical Advantage—A: 1 in. to 1 lb.
Percentage Efficiency—E: 1 in. to 10 %.

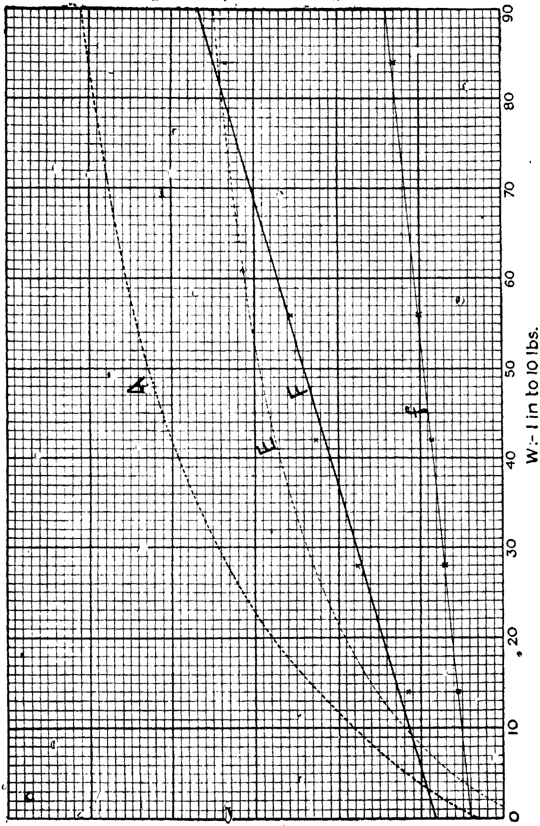

Fig. 59

Law of the Machine.—Since the effort curve is a straight line, showing graphically the relation

between F and W, this relation can be stated as a simple equation of the form:

$$F = aW + b,$$

where a and b are constants determined from the graph.

It may be seen from the graph in fig. 59 that a force of 4·2 lb. is required to overcome the resistance in the tackle with no load; this is the quantity b. The quantity a can now be found by choosing suitable values of F and W *from the graph*, and introducing these values into the equation. Thus, it is found that when W is 84 lb., F is 16·85 lb.; hence, since

$$\begin{aligned} F &= aW + b, \\ 16{\cdot}85 &= 84a + 4{\cdot}2, \\ 84a &= 16{\cdot}85 - 4{\cdot}2, \\ &= 12{\cdot}65. \\ \therefore a &= \frac{12{\cdot}65}{84}, \\ &= {\cdot}15, \\ \text{consequently, } F &= {\cdot}15W + 4{\cdot}2. \end{aligned}$$

This is the Mechanical Law of the machine. It must be borne in mind that this particular law is *not* applicable to *all* Weston blocks, but only to the one that was used for the experiment.

The law is used to correct the values of F given in Table V above. These corrected values appear in Table VI below, and are used to find the ratio $\frac{W}{F}$, the actual mechanical advantage at each load. The actual advantages are plotted as a graph at A in fig. 59, the curve showing that the advantage increases as the load gets heavier. It becomes

flat when the loads are heavy, which implies that the advantage tends to become constant as the load is increased.

TABLE VI

No. of Test.	W in lb.	Corrected F in lb.	$\frac{W}{F}$.
1	14	6·3	2·22
2	28	8·4	3·33
3	42	10·5	4·00
4	56	12·6	4·44
5	84	16·8	5·00

EFFICIENCY.—The efficiency at any particular load is the ratio of the work put in to the work got out. Alternatively, it is the ratio of the actual mechanical advantage to the theoretical advantage. Now:

$$\text{V.R.} = \frac{1}{\text{T.M.A.}};$$

$$\text{whence, T.M.A.} = \frac{1}{\text{V.R.}}$$

$$= \frac{1}{\frac{1}{14{\cdot}56}}$$

$$= 14{\cdot}56.$$

The efficiency at each load may therefore be calculated as in the following example, and the whole tabulated as in Table VII.

When the load is 84 lb., the M.A. is 5·00.

$$\text{Percentage efficiency} = \frac{\text{M.A.}}{\text{T.M.A.}} \times 100$$

$$= \frac{5{\cdot}00 \times 100}{14{\cdot}56} = 34{\cdot}35 \text{ per cent.}$$

Logarithms are very convenient for use in making such calculations. After the efficiency values have been found, the relation between W and the efficiency may be plotted as a graph, giving the line E in fig. 59.

TABLE VII

No.	Load.	Mechanical Advantage.	Per Cent Efficiency.
1	14	2·22	15·25
2	28	3·33	22·87
3	42	4·00	27·48
4	56	4·44	30 49
5	84	5·00	34·35

LOWERING EFFORT.—It is interesting also to obtain the relation between the load W and the force *f* necessary to *lower* the load. The spring balance S, fig. 58, must be attached to the lowering side of the chain, and experiments made to obtain *f* in exactly the same way as was done in obtaining F. These may be tabulated as in Table VIII below, and plotted on the graph as indicated by the line *f* in fig. 59.

TABLE VIII

No.	W.	*f.*	Corrected *f.*
1	14	$2\frac{3}{4}$	2·7
2	28	$3\frac{1}{2}$	3·48
3	42	$4\frac{1}{4}$	4·25
4	56	5	5·0
5	84	$6\frac{1}{2}$	6·55

If systematic experiments are being made on this

and similar mechanical appliances, the whole of the results may be conveniently arranged in one table as shown below.

TABLE IX

No.	W.	Experiment F	Corrected F.	$\frac{W}{F}$	Per cent Efficiency.	Experiment f.	Corrected f.
1	14	$5\frac{3}{4}$	6·3	2·22	15·25	$2\frac{3}{4}$	2·7
2	28	$8\frac{3}{4}$	8·4	3·33	22·87	$3\frac{1}{2}$	3·48
3	42	$11\frac{1}{4}$	10·5	4·00	27·48	$4\frac{1}{4}$	4·25
4	56	$12\frac{3}{4}$	12·6	4·44	30·49	5	5·0
5	84	$16\frac{3}{4}$	16·8	5·00	34·35	$6\frac{1}{2}$	6·55

Exercises with answers on p. 166.

CHAPTER XII

THE INCLINED PLANE

INCLINED PLANES.—Any plane surface lying at an inclination to the horizontal (or vertical) forms what is known in Mechanics as an *Inclined Plane*. The plane takes many forms in practice, and may be used for many different purposes. In general, however, it is used to produce mechanical advantage, whereby a comparatively small force can be made to overcome a larger resistance than it could do if applied directly. Inclined planes are therefore used in the lowering and raising of loads, in forcing machine parts together, e.g. keys and cotters; forcing them apart, e.g. wedges, &c. A screw may be regarded as a special form of inclined plane.

In the consideration of inclined planes it is convenient to neglect friction in the first instance, and to consider the plane as having no resistance to bodies

moving on it. Then, when the principles are understood, the results obtained can be modified to suit existing practical conditions.

There are then three distinct cases, diagrams of which are given in fig. 60.

I. The force may act parallel to the plane.
II. The force may act horizontally.
III. The force may act at any necessary angle to the plane.

In referring to inclined planes, it is convenient to use similar lettering, &c., in every case.

Fig. 61 shows the method employed in the following paragraphs. ABC is a triangle representing the essential dimensions of the inclined plane. AC or l is the length of the plane, say the slope of a road, the slope of a screw, or the slope on which any mechanical part moves on another mechanical part; AB, or b, is the base of the plane; and BC, or h, is the height of the plane.

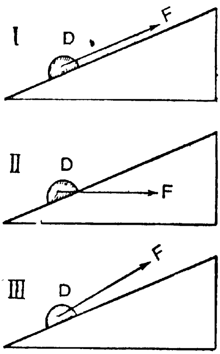

Fig. 60

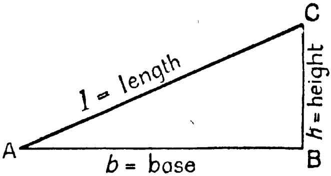

Fig. 61

Case I: The Force F is parallel to the Plane.—A body D, fig. 60 (say a sledge), is being pushed or pulled up an inclined plane (or road) AC by

a force (or horse) F. By the time the force has moved through a distance l (fig. 61), the body will have been raised through a height h. Then, by the Principle of Work;

$$F \times \text{distance moved by } F = W \times \text{distance moved by } W,$$

$$\text{i.e. } F \times l = W \times h.$$

$$\therefore F = \frac{Wh}{l}.$$

The Triangle of Forces may also be used to establish the relation between F and W. When the body is lying stationary upon the plane, it is kept in position by three forces:

1. W, the weight of the body acting downwards;
2. F, the force acting parallel to the plane; and
3. R, the reaction due to the weight of the body which, when friction is neglected, is at right angles to the plane.

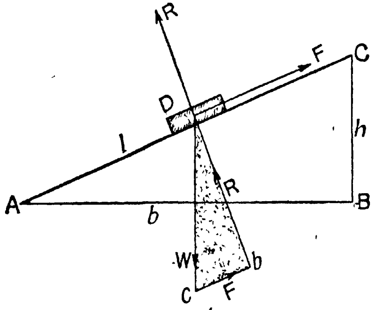

Fig. 62

Since the body is at rest, these three forces must pass through one point, and can thus be represented by the sides of a triangle. If particular values are

assigned, and the triangle of forces drawn, it will be found similar to the triangle *abc* in fig. 62. If triangle *abc* be further examined, it will be seen that it is similar in all respects to the plane diagram ABC; that is to say:

the side *ab*, representing the reaction R, corresponds to the side *b*;
the side *bc*, representing the force F, corresponds to the height *h*;
the side *ac*, representing the weight W, corresponds to the length *l*.

So that, by the Triangle of Forces:

$$W : F = ac : bc,$$

$$\text{i.e. } \frac{W}{F} = \frac{ac}{bc};$$

and, by the properties of similar triangles:

$$\frac{W}{F} = \frac{l}{h},$$

$$\text{whence } F = \frac{Wh}{l}.$$

This result corresponds exactly to that already found, so that by using the two methods, the Principle of Work, and the Triangle of Forces, the results are in complete agreement.

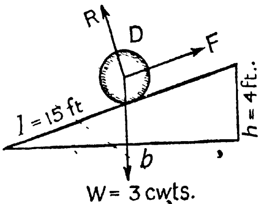

Fig. 63

Example 57.—A factory operative raises a cask D of tallow or cocoa-nut oil weighing 3 cwt. on to a car or lorry 4 ft. high, by rolling it up a plank 15 ft. long. Neglecting friction, what force will he exert, assuming that he pushes in

a line parallel to the plank, and what will be the pressure on the plank?

A diagrammatic view of the problem is given in fig. 63. As shown above:

$$F = \frac{Wh}{l}$$

$$= \frac{3\text{ cwt.} \times 4\text{ ft.}}{15\text{ ft.}}$$

$$= \frac{3 \times 112 \times 4}{15} = 89{\cdot}6\text{ lb.}\begin{cases}\text{the force}\\ \text{required.}\end{cases}$$

The pressure on the plank is the reaction due to the weight of the cask and contents.

Then, by the Triangle of Forces, and the properties of similar triangles:

$$W : R = l : b,$$

$$\text{i.e. } \frac{W}{R} = \frac{l}{b},$$

$$\text{hence } R = \frac{Wb}{l}.$$

Now W and l are known, but b is not; its value, however, may be found by noting that the plane diagram is a right-angled triangle, hence:

$$l^2 = b^2 + h^2,$$

$$\text{and } b = \sqrt{l^2 - h^2}$$

$$= \sqrt{(l + h)(l - h)}$$

$$= \sqrt{(15 + 4)(15 - 4)}$$

$$= \sqrt{19 \times 11}$$

$$= \sqrt{209}.$$

$$\therefore b = 14{\cdot}45\text{ ft.};$$

$$\text{consequently, } R = \frac{3 \times 112 \times 14{\cdot}45}{15}$$

$$\doteqdot 323{\cdot}7\text{ lb.}$$

CASE II: WHERE F IS HORIZONTAL.—Let AC, fig. 64, represent the inclined plane up which a body D is being raised by the action of a force F acting horizontally. By the time the body has been raised through a height BC or h, it will have travelled along the length of the plane AC or l, and the force F will

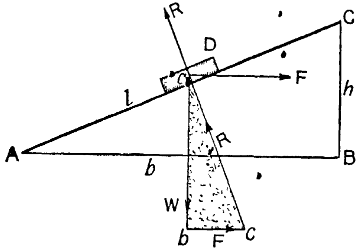

Fig. 64

have moved through a distance AB or b. Then, by the Principle of Work:

$$F \times \text{distance moved by } F = W \times \text{distance moved by } W,$$

$$\text{or } F \times b = W \times h,$$

$$\text{whence } F = \frac{Wh}{b}.$$

If particular values are assigned, a triangle of forces may be drawn as was done in Case I. The force triangle will be different in formation, but it will still be *similar* to the plane diagram, i.e. the triangle abc in fig. 64 is similar in all respects to the triangle ABC, thus:

side ab corresponds to W, and is proportional to AB or b;

side bc corresponds to F, and is proportional to BC or h;

side ac corresponds to R, and is proportional to AC or l.

Thus, by the Triangle of Forces:

$$\frac{F}{W} = \frac{bc}{ab};$$

and by the properties of similar triangles:

$$\frac{F}{W} = \frac{h}{b},$$

$$\text{whence } F = \frac{Wh}{b}.$$

This agrees with the relation already found by the Principle of Work; the relation between F and R, and also between W and R, may be established by similar means.

Example 56.—An inclined plane rises 1 ft. for every 15 ft. of its length. Neglecting friction, find the force required to pull a body weighing 1 cwt.

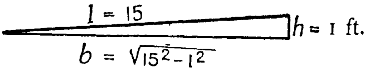

Fig. 65

up the plane if the direction of the force is horizontal; find also the reaction in the plane due to this load.

As shown above:

$$F = \frac{Wh}{b}$$

$$= \frac{112 \text{ lb.} \times 1 \text{ ft.}}{b},$$

$$\text{and } b = \sqrt{15^2 - 1^2} \text{ (see fig. 65)},$$

$$\text{hence } F = \frac{112 \times 1}{\sqrt{15^2 - 1^2}}$$

$$= \frac{112}{\sqrt{225 - 1}} = \frac{112}{\sqrt{224}}$$

$$= \frac{112}{14{\cdot}96} \doteqdot 7{\cdot}48 \text{ lb.} \left\{ \begin{array}{l} \text{the force} \\ \quad \text{required.} \end{array} \right.$$

By the Triangle of Forces and the properties of similar triangles

$$R : W = l : b,$$

$$\frac{R}{W} = \frac{l}{b}.$$

$$\therefore R = \frac{Wl}{b}$$

$$= \frac{112 \times 15}{14{\cdot}96}$$

$$\doteqdot 112{\cdot}3 \text{ lb.}$$

Case III: When F is at ANY Angle.—Generally speaking, this case is slightly more difficult of solution than the two former cases. In practically every case, however, the triangle of forces may be applied, and where graphical methods are not considered sufficiently accurate, trigonometrical methods may be used.

As already pointed out, the relations expressed above do not take account of frictional and other resistances; the following example shows how the results obtained must be modified to suit particular conditions.

Example 59.—Trolley cars, electrically driven by motor and accumulator, are used in a large textile establishment to convey bales of raw material and finished goods over the lines of a works railway. The trolley weighs 10 cwt. net, and 30 cwt. when

fully loaded. If the tractive resistance on the level is equal to 20 lb. per ton, and the efficiency of motor and drive is 75 per cent, what horse-power will be required to propel the car at a speed of 10 miles per hour—

1. On the level, and
2. Up a gradient of 1 in 15?

(1) On the level:—

$$\text{Useful H.P.} = \frac{RS}{33000} = \frac{20 \text{ lb. per ton} \times 1\tfrac{1}{2} \text{ tons} \times 10 \text{ ml. per hr.}}{33000}$$

$$= \frac{(20 \times 1{\cdot}5) \text{ lb.} \times 10 \times 5280 \text{ ft. per ml.}}{33000 \times 60 \text{ min. per hr.}}$$

$$= \frac{20 \times 1{\cdot}5 \times 10 \times 5280}{33000 \times 60}.$$

The efficiency of the motor and drive is only 75 per cent, so that this useful horse-power must be increased in the ratio of 100 per cent to 75 per cent. Consequently

$$\text{Total H.P.} = \frac{20 \times 1{\cdot}5 \times 10 \times 5280}{33000 \times 60} \times \frac{100}{75}$$

$$= \frac{176}{165} = 1{\cdot}067 \text{ H.P.}$$

It should be kept in mind that this would be the minimum horse-power required to *keep the trolley running* under the given conditions. A motor of a higher power would actually be used in order to be able to start the trolley *from rest*, i.e. to overcome the inertia resistances.

(2) Up a gradient of 1 in 15.

N.B.—A gradient of 1 in 15 means a rise of 1 ft. in 15 ft. of the *plane* (see fig. 65, which may be taken to represent the required gradient). In moving the trolley up the incline, the motor exerts a tractive

effort of 20 lb. per ton, necessary for rail resistance alone; in addition, it must exert a further force capable of *lifting* the whole weight of the trolley through 1 ft. in every 15 ft. it travels. It will also be evident that the force must be parallel to the plane. Now: Tractive effort to overcome rail resistance,

$$F_t = 20 \text{ lb.} \times 1\tfrac{1}{2} \text{ tons} = 30 \text{ lb.}$$

Since the force is parallel to the plane, the lifting effort is:

$$F_l = \frac{Wh}{l} \quad \text{(See Case I, page 129.)}$$

$$= \frac{1\tfrac{1}{2} \text{ tons} \times 1 \text{ ft.}}{15 \text{ ft.}}$$

$$= \frac{3360 \times 1}{15} = 224 \text{ lb.}$$

$$\text{Total F} = F_t + F_l$$

$$= 30 + 224 = 254 \text{ lb.}$$

$$\text{Useful H.P.} = \frac{R \times S}{33000} = \frac{254 \times 10 \times 5280}{33000 \times 60}.$$

But since the motor, &c., has an efficiency of only 75 per cent—

$$\text{Total H.P.} = \frac{254 \times 10 \times 5280}{33000 \times 60} \times \frac{100}{75}$$

$$= 9{\cdot}031 \text{ H.P.}$$

Again, this result allows no margin for starting the trolley from rest; it shows, however, the decided influence that gradients have in increasing the required horse-power, and the consequent necessity of reducing gradients to a minimum wherever possible. Ball and roller bearings and other anti-frictional devices will undoubtedly reduce the *tractive* effort, but nothing can reduce the *lifting* effort, since it

depends on the weight, and this in turn is dependent on the force of gravity alone.

Exercises, with answers, on p. 167.

CHAPTER XIII

THE SCREW

It has already been pointed out that the screw—many forms of which are doubtless familiar to all who have anything to do with machinery—may be regarded as a special form of inclined plane. Obtain a small cylinder of any material, and cut out a

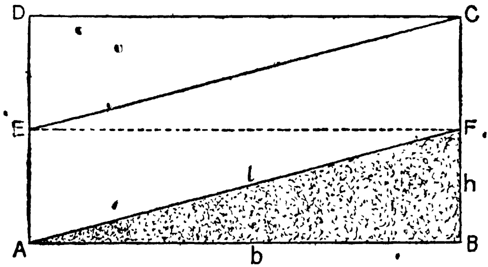

Fig. 66

rectangle from paper, so that, when the latter is wrapped round the cylinder, its edges will meet, and the paper itself cover completely the curved surface of the cylinder. ABCD, fig. 66, is such a rectangle; it is capable of covering the curved surface of a cylinder, the dimensions of which are:

height, AD; circumference, AB.

Bisect the sides AD and BC in E and F respec-

tively, and join AF and EC. Re-wrap the paper on the cylinder, and see that the line AFEC now forms a continuous spiral of constant inclination round the curved surface of the cylinder. The line may be substituted by a V-shaped rib, in which case a close approximation to the thread of an ordinary screw will be obtained, and the spiral AFEC would in reality represent two complete threads.

Cut off the triangle ABF, shown stippled in fig. 66; this may evidently be regarded as an inclined plane equivalent in length to one complete thread. Suppose now that a nut is being tightened on a bolt. A force is applied, usually through some form of lever such as a nut-key or spanner, and the nut is caused to make one complete revolution. In doing so, the force will have moved through a distance equal to the circumference of the screw, and corresponding to the line AB in fig. 66, and to the base b of the inclined plane. In the same time, the nut will have advanced along the thread of the screw and overcome frictional and other resistances through a distance equal to the pitch of the screw (distance between successive threads) corresponding to the line BF in fig. 66, and to the height h of the inclined plane. Now, by the Principle of Work:

F $\times$ distance moved by F $=$ W $\times$ distance moved by W,

$\therefore$ F $\times$ circumference of screw $=$ W $\times$ pitch of screw;

$$\text{or} \qquad F \times b = W \times h,$$

$$\text{whence} \qquad F = \frac{Wh}{b}.$$

This is the same result as was obtained in Case II of the Inclined Plane, p. 133, when the force F is

acting horizontally or parallel to the base. Screws may therefore be treated, so far as the relations between F, W, and R are concerned, as inclined planes of Case II. The force is seldom applied directly at the screw thread; as indicated above, some form of lever is generally used, so that, more correctly speaking, a screw may be regarded as a combination of the inclined plane and the lever. If it is assumed that a lever of effective length L is used, the relation between F and W may be more conveniently expressed in terms of this length L, and in the ordinary dimension of the screw-thread, viz. the pitch. Thus, since

$$F = \frac{Wh}{b},$$

$$\therefore Fb = Wh;$$

but b is the *circumferential* distance through which F moves, and h is the pitch of the screw or the distance through which the resistance W is overcome. Therefore:

$$F \times 2\pi L = W \times p$$

where L is the effective length of the lever, and p is the pitch of the screw.

$$\therefore F : W = p : 2\pi L,$$

$$\text{and } F = \frac{Wp}{2\pi L}.$$

By using similar mathematical means, expressions may be found giving the relations between F and R, and between W and R, when such are necessary.

Example 60.—A holding-down bolt for a heavy loom has a screw thread of $\frac{1}{8}$ in. pitch, and has an efficiency of 25 per cent. Find the resistance overcome which is equal to the gripping pressure of the nut, when it is screwed up with a force of 30 lb. at the end of a spanner with an effective length of 18 in.

By the Principle of Work, and neglecting friction:

$$F \times 2\pi L = Wp,$$

$$\text{whence } W = \frac{F \times 2\pi L}{p}.$$

If the efficiency is only 25 per cent, F will only be able to overcome 25 per cent of this resistance, i.e. the gripping pressure will be:

$$W = \frac{F \times 2\pi L}{p} \times \frac{25}{100}$$

$$= \frac{30 \times 2 \times 3{\cdot}14 \times 18}{{\cdot}125} \times \frac{25}{100}$$

$$= 6782{\cdot}4 \text{ lb.}$$

In other words, a resistance of 6782·4 lb. is overcome by a force of 30 lb.; this is sufficient to show the enormous mechanical advantage obtained by using a screw. The student will find it interesting and instructive to work out the theoretical and actual mechanical advantages under the given conditions.

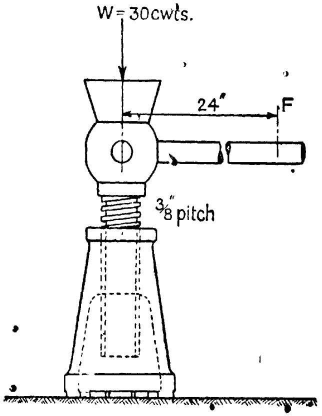

Fig. 67

Example 61.—A common screw-jack, such as is illustrated in fig. 67, is employed to hold up one gable of a cloth mangle while repairs are being executed. The lever used has an effective length of 24 in., the screw is

$\frac{3}{8}$ in. pitch, and its efficiency is 30 per cent. If the proportion of the total load being sustained is 30 cwt., find what force is used in lifting it.

By the Principle of Work, and neglecting friction, we have:

$$F \times 2\pi L = Wp,$$

$$\text{whence } F = \frac{Wp}{2\pi L}$$

$$= \frac{30 \text{ cwt.} \times \frac{3}{8} \text{ in.}}{2 \times 3{\cdot}14 \times 24 \text{ in}}$$

$$= \frac{30 \times 112 \times {\cdot}375}{2 \times 3{\cdot}14 \times 24}$$

$$= \frac{105}{12{\cdot}56} \doteq 8{\cdot}36 \text{ lb.}$$

But the efficiency is only 30 per cent, therefore F must be increased in the ratio of 100 to 30, that is:

$$F = \frac{8{\cdot}36 \times 100}{30} \doteq 27{\cdot}87 \text{ lb.}$$

Exercises, with answers, on p. 168.

CHAPTER XIV

POWER TRANSMISSION: BELTS AND GEARS

In textile mills and factories of all kinds, the power required to drive the various machines is usually developed by some form of steam-engine, and either of the reciprocating or the turbine type. This power has then to be distributed over the mill or factory, and

many, and varied are the means of transmission. Ropes, belts, gears, chains, electricity, &c., are used, each having certain advantages under particular conditions.

TYPES OF DRIVE.—Ropes and belts are wrapped partially round pulleys, the peripheries of which are shaped to suit, and the drive is said to be by *wrapping contact.* Gears and chains drive by what is known as *rolling contact.* When electricity is used, it is distributed by cables to suitably-placed motors, and the

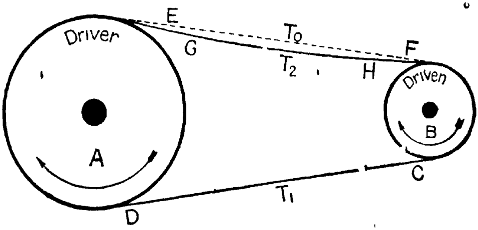

Fig. 68

drive between motor and machine is then one of the four above-mentioned types; in certain cases a separate motor may be coupled directly to each machine, and then it is termed an individual drive.

BELTS AND ROPES.—Suppose that A, fig. 68, is a large pulley, 10 ft. diameter, and running at 100 r.p.m. Its surface speed or relative motion may be transmitted by means of a belt to a second pulley B, 5 ft. diameter. Provided that there is no slip, the speed of the belt would be

$$\pi \times 10 \text{ ft.} \times 100 \text{ r.p.m.} = 3141{\cdot}6 \text{ ft. per minute.}$$

This speed must also be the surface speed of the pulley A and that of the pulley B, but since B has a smaller

circumference than A, it must make a larger number of revolutions per minute. Its speed must be:

$$\frac{3141 \cdot 6 \text{ ft. per minute}}{\pi \times 5 \text{ ft.}} = 200 \text{ r.p.m.}$$

Now the diameter of A is 10 ft., and its speed 100 r.p.m., and the diameter of B is 5 ft., and its speed 200 r.p.m.; that is to say, if the diameter is halved, the revolutions per minute are doubled.

Although taken for a particular case, the reasoning is perfectly general, and it may be concluded that the speeds of pulleys directly connected by ropes or belts are inversely proportional to their diameters. Where pulley speeds are alone concerned, it is not necessary in practice to find the actual belt speed, since, as is shown above:

$$\text{Diam. of A} : \text{diam. of B} = \text{r.p.m. of B} : \text{r.p.m. of A},$$

$$\text{or,}\quad \frac{\text{diameter of A}}{\text{diameter of B}} = \frac{\text{r.p.m. of B}}{\text{r.p.m. of A}},$$

$$\text{whence, r.p.m. of B} = \frac{\text{diam. of A} \times \text{r.p.m. of A}}{\text{diam. of B}}.$$

Substituting the figures used above:

$$\text{r.p.m. of B} = \frac{10 \text{ ft.} \times 100 \text{ r.p.m.}}{5 \text{ ft.}} = 200 \text{ r.p.m.},$$

as before found.

Example 62. — A flax roving frame has 24-in.-diameter pulleys, which are driven by a belt from a 30-in.-diameter pulley fixed to a line-shaft running at 200 r.p.m. Find the speed of the roving frame pulleys:

$$\begin{aligned}\text{Pulley speed} &= \text{r.p.m. of shaft} \times \frac{\text{diam. of driving pulley}}{\text{diam. of driven pulley}}\\ &= 200 \text{ r.p.m.} \times \frac{30 \text{ in.}}{24 \text{ in.}}\\ &= 250 \text{ r.p.m.}\end{aligned}$$

The result is not strictly correct, since the method used ignores the thickness of the belt. It is obvious that a belt has thickness, and that this thickness increases the *effective* diameter of the pulleys; the effective diameter must be measured from centre to centre of the belt wrapped round the pulley, and in all cases will be equal to the diameter of pulley plus the thickness of the belt.

Example 63.—The conditions are similar to those in Example 62, but a belt $\frac{3}{8}$ in. thick is employed. Calculate the speed of the small or driven pulley, allowing for this belt thickness:

Driven pulley speed

$$= \text{r.p.m. of shaft} \times \frac{\text{effective diam. of driving pulley}}{\text{effective diam. of driven pulley}}.$$

Driven pulley speed

$$= 200 \times \frac{30 \text{ in.} + \frac{3}{8} \text{ in.}}{24 \text{ in.} + \frac{3}{8} \text{ in.}} = 200 \times \frac{30{\cdot}375}{24{\cdot}375}$$

$$= 249{\cdot}23 \text{ r.p.m.}$$

It will be appreciated that the difference in the two results is small; it is not usual, for this reason, to take the belt thickness into consideration unless in cases of very small diameter pulleys. A further reason is that in either case the result is only approximate, due to the interference of slip.

Belt Slip.—Belts or ropes may slip on pulleys for various reasons:—

1. When the mechanical design is unsound. This generally resolves itself into a case of the belt being too weak in some respect for its particular duty. This slip may be reduced to a minimum by increasing the belt speed, by increasing the diameters of the pulleys, or by using a wider belt.

2. The above might be described as avoidable slip, but it will be seen that a certain amount of slip, varying in practice from 2 to 3 per cent, is unavoidable. Thus:

(*a*) Suppose the belt instanced in Examples 61 and 62 is $\frac{3}{8}$ in. thick, it is evident that the surface next to the face of the pulley must move at a slower rate than the outer surface of the belt. As this cannot occur for any length of time without involving the destruction of the belt, the latter must slip on the face of the pulley in order to adjust itself. It will be understood that this adjustment is continuous and not periodic.

(*b*) As will be discussed later, there is a difference in the tension of the driving side of a belt and the tension of the slack side of a belt. There is actually a greater tension in the driving side than in the slack one, so that the stretch due to the pull in the slack side must be less than the stretch in the driving side. It follows that a slightly longer length of belting is received from the driven pulley and delivered to the driving pulley than is delivered from the driving pulley and received on the driven pulley. This continuous variation in the belt length is compensated for by slip, and this particular kind of slip is termed *creep*.

It is a good practice, therefore, to allow for this unavoidable slip by increasing the calculated diameter of the driving pulley, or by decreasing that of the driven pulley, by 2 to 3 per cent.

In the great majority of textile calculations the above considerations are ignored; nevertheless, it is well to know that the results obtained are approximations, and to know the reasons for their being approximations.

Example 64.—The fly-wheel of an engine drives a factory lighting dynamo by means of countershafts, pulleys, and belts, as indicated in fig. 69. Find the velocity ratio between the fly-wheel of the engine and

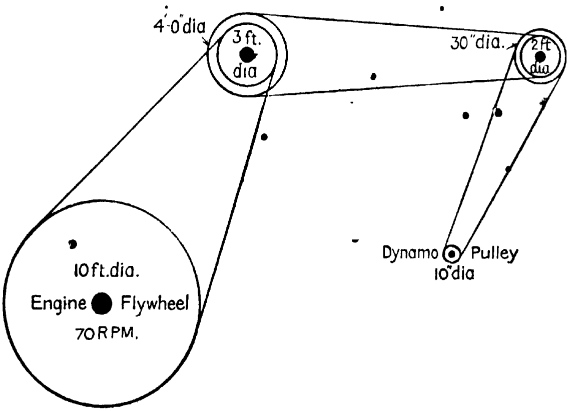

Fig. 69

the dynamo, the speed of the dynamo ignoring slip, and the speed when 3 per cent is allowed for slip.

$$\text{Velocity Ratio} = \frac{\text{product of all driven pulleys}}{\text{product of all driving pulleys}}$$

$$= \frac{10 \text{ in.}}{30 \text{ in.}} \times \frac{2 \text{ ft.}}{4 \text{ ft.}} \times \frac{3 \text{ ft.}}{10 \text{ ft.}} = \frac{1}{20},$$

i.e. 1 rev. of engine fly-wheel = 20 revs. of dynamo; hence:

$$\text{speed of dynamo} = 70 \text{ revs.} \times 20 = 1400 \text{ r.p.m.},$$

$$\text{or, dynamo speed} = 70 \times \frac{10 \text{ ft.}}{3 \text{ ft.}} \times \frac{4 \text{ ft.}}{2 \text{ ft.}} \times \frac{30 \text{ in.}}{10 \text{ in.}}$$

$$= 1400 \text{ r.p.m.}$$

If 3 per cent is lost in slip, the actual speed will be 97 per cent of 1400 r.p.m. Thus:

$$\left(\frac{100 - 3 \text{ per cent}}{100}\right) 1400 = \frac{97}{100} \text{ of } 1400 = 1358 \text{ r.p.m.}$$

Tension in Belts and Ropes. When a belt or rope is placed over a pair of pulleys such as those shown in fig. 68, it must be put on with sufficient initial tension to allow it to grip firmly the pulley surfaces. Let this initial tension be T_0; the belt will then occupy a position similar to the line CDEF. When pulley A, the driver, begins to move, it will momentarily stretch or pull the under side or length of the belt until the tension is sufficiently great to overcome the resistance in B, the driven pulley. The initial tension T_0 is thereby increased to T_1. Simultaneously, the tension in the upper length of the belt will decrease by a corresponding amount, and whereas the *under* or *driving side* will become tight, the *upper* or *following side* will become slack. The belt will then run in a path similar to CDGH. Since the upper side of the belt has changed its path from EF to GH, the tension in the driven side may now be considered to have changed from T_0 to T_2 as indicated. It is evident that the pull available for actual driving must be the *difference* between these two tensions, i.e. it must be $T_1 - T_2$.

If P represents the effective working pull in the belt, then:

$$P = T_1 - T_2.$$

From the nature of the problem, T_1 must be greater than P or T_2, hence, in calculating belts for particular duties, it is T_1 that must be considered rather than P.

T_1 and T_2 have been measured by a special form

of dynamometer, and the ratio $\frac{T_1}{T_2}$ found to vary between $\frac{10}{3\frac{1}{2}}$ and $\frac{10}{7}$, a common value being $\frac{10}{4}$.

Example 65.—A factory lineshaft drives a single opener requiring 5 H.P. by means of a belt. The driving pulley is 30 in. diameter and runs at 200 r.p.m. The belting used is such that 1 in. width can stand a working pull of 45 lb. If the tensions in the tight and slack sides are as 10 to 4, find a suitable width for the belt. ($\pi = 3{\cdot}14$.)

Let P = the effective pull in the belt in pounds required to overcome the resistance R.

$$\text{Now,}\quad \text{H.P.} = \frac{\text{RS}}{33000}$$

$$\therefore\quad \text{H.P.} = \frac{\text{PS}}{33000},\quad \text{since R} = \text{P},$$

$$\text{hence,}\quad \text{P} = \frac{33000 \times \text{H.P.}}{\text{S}}$$

$$= \frac{33000 \times 5}{200 \times 2{\cdot}5 \text{ ft.} \times 3{\cdot}14}$$

$$\fallingdotseq 105{\cdot}1 \text{ lb.},$$

$$\text{but,}\quad \text{P} = \text{T}_1 - \text{T}_2,$$

$$\text{and}\quad 10 - 4 = 6,$$

$$\text{whence,}\quad \text{T}_1 = \frac{105{\cdot}1 \times 10}{6}$$

$$\fallingdotseq 175{\cdot}2 \text{ lb.}$$

$$\text{Width of belt} = \frac{175{\cdot}2 \text{ lb. total pull}}{45 \text{ lb. pull per in. width}}$$

$$= 3{\cdot}89 \text{ in., say a 4-inch belt.}$$

Example 66.—A calender absorbs 12 H.P., and receives motion from a driving pulley 4 ft. diameter running at 65 r.p.m. The belting used can take a working pull of 220 lb. per square inch. If the belt

is $\frac{1}{2}$ in. thick (a heavy double belt) and the tensions on tight and slack sides are as 10 to $4\frac{1}{2}$, find the width of belt required. (Use $\pi = \frac{22}{7}$.)

$$\text{H.P.} = \frac{RS}{33000}, \text{ and}$$

$$P = R = \frac{33000 \times \text{H.P.}}{S}$$

$$= \frac{33000 \times 12}{4 \text{ ft.} \times \pi \times 65} = \frac{33000 \times 12 \times 7}{4 \times 65 \times 22}$$

$$= 484 \cdot 61 \text{ lb. the effective pull.}$$

$$\text{Now, } P = T_1 - T_2,$$

$$\text{and } 10 - 4\tfrac{1}{2} = 5\tfrac{1}{2},$$

$$\therefore T_1 = \frac{P \times 10}{5\frac{1}{2}} = \frac{484 \cdot 61 \times 10}{5 \cdot 5}$$

$$= 881 \cdot 1 \text{ lb. pull on tight side.}$$

$$\frac{881 \cdot 1 \text{ lb. total pull}}{220 \text{ lb. pull per sq. in.}} \doteqdot 4 \text{ sq. in.},$$

$$\text{and, } \frac{4 \text{ sq. in.}}{\frac{1}{2} \text{ in. belt}} = 8 \text{ in. wide belt.}$$

Gears. — Gears are used to transmit power and motion where exact speed ratios are necessary, and where the distance between the centres of the driving and driven elements is too short to permit the use of ropes or belts. They form necessary and important parts of practically all textile machinery.

The number of teeth in a gear of any description bears an exact and definite relationship to the pitch diameter (see Chap. XI, Woodhouse and Brand's *Textile Mathematics I.*). The speeds of all the gears in any train are inversely proportional to their diameters, and consequently to the number of teeth. Calculations concerning the speed relations of gears are therefore worked out on exactly the same

principles as those concerning pulleys, except that the numbers of teeth are used in place of the diameters. The numbers of teeth in the sprockets are also used in the case of chain drives.

Example 67.—The crank shaft of a loom runs at 180 r.p.m.; a pinion of 24 teeth keyed on it gears with a wheel of 48 teeth on the bottom or tappet shaft. Find the speed of the bottom shaft.

$$\frac{\text{crank shaft speed}}{\text{bottom shaft speed}} = \frac{\text{bottom shaft wheel}}{\text{crank shaft pinion}},$$

hence:

$$\text{speed of bottom shaft} = \text{crank shaft speed} \times \frac{\text{crank shaft pinion}}{\text{bottom shaft wheel}}$$

$$= 180 \times \frac{24}{48} = 90 \text{ r.p.m.}$$

Direction of Rotation.—In determining the train of gear to suit a particular set of conditions, the

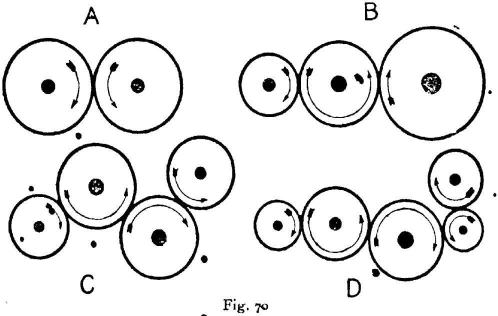

Fig. 70

direction of rotation is just as important as the speed, and in certain cases more important. Consider, for instance, the various trains of gear shown in fig. 70. The diagrams at A, B, C, and D represent four

simple trains of gear, consisting respectively of 2, 3, 4, and 5 wheels. In each case the wheel on the left is taken as the driver, and the wheel on the right as the driven, and all the drivers rotate clockwise. It will be observed that where the number of wheels is *even* (i.e. 2, 4, &c., as in A and C), the driven wheel runs in the *contrary* direction to the driver; but where the number of wheels is *odd* (i.e. 3, 5, &c., as at B and D), the driven wheel runs in the *same* direction as the driver.

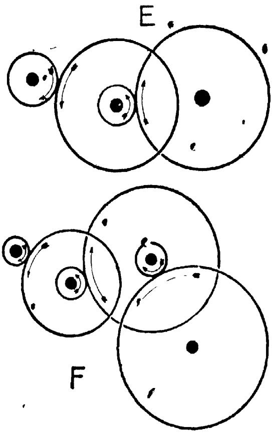

Fig. 71

IDLE OR INTERMEDIATE WHEELS.—The wheels between the driver and driven wheels in such trains of gear as B, C, and D are variously termed carrier wheels, single intermediate wheels or idle wheels. It is important to note that, while the number of such wheels influences the direction of rotation of the driven wheel, the number of teeth in the various wheels has no influence upon the speed relation of the driver and driven wheels. In other words, all single intermediate wheels may be ignored so far as wheel speed calculations are concerned.

COMPOUND TRAINS.—Fig. 71 shows two examples of *compound* trains of gear, consisting of two lines of gear. The gear at E contains one double intermediate or compound wheel, while that at F has two

such wheels. In example E, note that though the number of wheels is even (4), the number of axes or arbors is *odd* (3), and that the direction of the first driver and of the last driven wheel is the *same*. Again, in example F, the number of wheels is still even (6), but the number of axes or arbors is also *even*, while the direction of the last driven wheel is *opposite* to that of the first driver.

The general inference, therefore, is that, when the number of axes is *odd*, the directions of the two extreme wheels are *similar*; and when the number of axes is *even*, the directions of the two extreme wheels are *opposite*.

SPEED RELATIONS OF CARRIERS.—*Single* intermediate wheels do *not* affect the speed relations, since a movement of one tooth of the driver results in a movement of one tooth in all the members of the same train, as exemplified at A, B, C, and D, fig. 70. But with double intermediates or compound wheels, such as those at E and F, fig. 71, the same rule does not apply.

Suppose, for instance, that in the train E, fig. 71, the double intermediate wheel has 90 teeth on the large one and 30 teeth on the small one. A movement of the driver of one tooth will cause the 90-tooth wheel to advance one tooth, i.e. $\frac{1}{90}$ of a revolution. But, since the 30-tooth wheel is compounded with the 90-tooth wheel, the former will also advance $\frac{1}{90}$ of a revolution $= \frac{1}{90}$ of 30 teeth $= \frac{1}{3}$ tooth, and, in consequence, the driven wheel rotates the equivalent of $\frac{1}{3}$ of a tooth. That is to say, a movement of *one* tooth of the driver results in the equivalent of a *third* of a tooth movement of the driven. Hence, it is obvious that all such compound wheels must be taken into consideration when calculating speeds.

Example 68.—Diagram C, fig. 70, may be taken as the gear between the cylinder of a card (running at 180 r.p.m.) and its drawing roller. The cylinder pinion (driver) has 60 teeth; the intermediate wheels have 80 and 90 teeth respectively, while the drawing roller wheel (driven) has 72 teeth. Find the speed of the drawing roller.

$$\text{Drawing roller speed} = \text{cylinder speed} \times \frac{\text{cylinder pinion}}{\text{drawing roller wheel}},$$

$$= 180 \times \frac{60}{72} = 150 \text{ r.p.m.}$$

The intermediate wheels are not considered in the calculation, because they are single or carrier wheels only.

Example 69.—Diagram F, fig. 71, may be regarded as the gear between the cylinder pinion of the same card and its feed roller. The intermediate wheels have 80 and 30 teeth, and 120 and 20 teeth, and the feed roller wheel has 120 teeth. Find the speed of the feed roller.

$$\text{Speed of feed roller} = 180 \times \frac{60}{80} \times \frac{30}{120} \times \frac{20}{120}$$

$$= \frac{45}{8} = 5{\cdot}625 \text{ r.p.m.}$$

In this case the intermediate wheels are considered, because they are double or compound wheels.

Exercises, with answers, on p. 169.

EXERCISES

Chapter I, pp. 7–14

1. Write down an exact definition of the term *matter*, with special reference to Mechanics.

2. Give an exact definition of the term *force*, as used in Mechanics.

3. What is the unit of force? Why is it only approximately correct?

4. State the elements of a force, and show by an example how a force may be completely represented by a straight line.

5. What is a force scale, and for what purpose is it used? Illustrate your answer by a typical example.

6. Represent by means of straight lines:
   (*a*) A vertical force, acting due north, of $3\frac{1}{2}$ tons to a scale of 1 in. to 1 ton; and
   (*b*) A horizontal force, acting towards the left, of 10 cwt. to a scale of 1 in. to 4 cwt.

7. Represent by means of straight lines:
   (*a*) A force of 8 lb. acting obliquely downwards at an angle of 45° to a scale of 1 in. to 2 lb.; and
   (*b*) A force of 50 lb. acting obliquely upwards at 60° to the horizontal to a scale of 1 in. to 20 lb.

8. Give definitions distinguishing between the term *Statics* and the term *Dynamics*.

Chapter II, pp. 14–34

1. Give definitions which distinguish between *force* and the *work done* by a force. In what units is each respectively measured?

2. Give a clear description of the terms *foot-pound* and *horse-power*, and show by examples how each is used.

3. Distinguish between a "Force of 1 lb." and a "Weight of 1 lb.", and between the "Power of a horse" and "One horse-power".

4. A dyehouse pump is employed to raise 200 gall. of liquor (10 lb. per gallon) through a height of 16 feet. Find the number of units of mechanical work expended in the process.

Ans. 12,000 ft.-lb..

5. In fitting a key for a loom crank-pinion, the fitter makes 30 double strokes of the file per minute. Each stroke of the file is 10 in. long, the force during the forward stroke being 30 lb., and during the back stroke 3 lb. At what H.P. is he working?

Ans. $\frac{1}{40}$ H.P.

6. The water which supplies the power for driving a small woollen mill falls 25 ft. If 20 per cent of the water goes to waste, what weight of water must pass over the fall per minute in order that the wheel may develop 20 H.P. *Ans.* 33,000 lb.

7. On a diameter XY of 5 in. describe a semicircle. Consider this as a diagram of work done by a variable force moving along XY. If the force scale is 1 in. to 20 lb., and if XY represent 5 ft., find the average force throughout the movement, and the total work done by the force in moving from X to Y.

Ans. 39·27 lb. 196·35 ft.-lb.

8. A steam wagon engaged in the transport of textile material between the works and the railway depot weighs 10 tons when fully loaded. The road resistance is 30 lb. per ton. The engine cylinder is 8 in. dia., the stroke 15 in., the mean effective pressure 33 lb. per square inch, and the speed 80 r.p.m. Find the H.P. developed, and the travelling speed in miles per hour along a level road. *Ans.* 10·05 H.P. 12·56 m.p.h.

9. A centrifugal pump is employed as part of a condensing-plant in raising 6 c. ft. of water per second to a height of 20 ft. If its efficiency is 50 per cent, what H.P. must be supplied to drive it? *Ans.* 27·28 H.P.

10. A single-cylinder double-acting steam-engine, used for driving a large finishing-house, has a cylinder 36 in. diameter, a stroke of 8 ft., and runs at 60 r.p.m. Find the indicated horse-

power if an indicator diagram gives the mean effective pressure as 41·5 lb. per square inch. $\pi = 3 \cdot 1416$. *Ans.* 1229 H.P.

11. A hydraulic packing-press has a 6-in.-diameter ram; water is supplied to the press from a pump with 4 plungers, each $2\frac{1}{4}$ in. diameter, with 3 in. stroke. Neglecting frictional and other losses, find the average rate at which the pump works in ft.-lb. per minute if it makes 100 working strokes per minute, and the press exerts a total force of 72 tons. *Ans.* 567,000 ft.-lb.

12. A factory operative, walking at 2 miles per hour, can exert a pushing force of 50 lb., and at 4 miles per hour a force of 25 lb. Find the total load he can move at these speeds on a road where the resistance is 55 lb. per ton.

Ans. 2036·4 lb. and 1018·2 lb.

Chapter III, pp. 34–38

1. Explain the meaning of the terms "Component", "Resultant", and "Equilibrant" as applied to a system of forces.

2. In regard to fig. 3, suppose the operative exerts a force of 25 lb. in pulling the basket of pirns along the floor, and that the cord makes an angle of 30° with the horizontal. Find the force actually exerted in moving the basket horizontally along the floor, and that exerted in tending to lift the basket vertically from the floor. *Ans.* 21·65 lb. 12·5 lb.

Chapter IV, pp. 38–44

1. Define the centre of gravity of a body, and show how it may be found experimentally in the case of an irregular body.

2. From a piece of cardboard or from a thin sheet of metal (such as tinned steel) cut out a semicircle 5 in. diameter. Calculate the position of its centre of gravity and mark this position on the figure. Check your result by attempting to balance the semicircle on the thin edge of a steel straight-edge.

Ans. 1·061 in. from centre of base.

3. Use your knowledge of the position of the centre of area of a circle to calculate the weight of a steel ring of circular section, given that the internal diameter is 4 in., and that the ring is made from $\frac{3}{8}$ in. round steel weighing $\frac{3}{4}$ lb. per foot.

Ans. ·859 lb. or 13·74 oz.

4. A T-shaped cast-iron rail is used in a certain winding-machine, and it is desired to know the position of its centre of gravity. The top cross-piece is 5 in. long by $\frac{3}{4}$ in. deep, and the stem is 6 in. deep by $\frac{5}{8}$ in. wide, while the rail is 6 ft. long. Find the centre of gravity.

Ans. Centre of gravity is at the intersection of the central line of the stem and the middle point of the plane adjoining the stem.

5. (This should be left until Chapter VII on Moments has been read.) The back wall of a factory building, built on an incline, is 150 ft. long, and trapezoidal in section, being 18 in. thick at the top and 3 feet thick at the bottom. The height of the wall is 16 ft., and the centre of gravity of the section is 7 ft. from the base. If the masonry weighs 130 lb. per c. ft., find what force, applied along a line 2 ft. from the top, would be necessary to overturn the wall, i.e., to cause it to pivot about its base.

Ans. 351,000 lb.

Chapter V, pp. 44–56

1. Two forces, P and Q, of 12 lb. and 16 lb. respectively, meet at a point and enclose an angle of 45°. Find the resultant R. Use the formula (which has not been proved in this work) $R^2 = P^2 + Q^2 + 2\,PQ\cos\theta$, where θ in this case is 45°. Check by any graphical method. $\sqrt{2} = 1{\cdot}41$.

Ans. $R = 24{\cdot}7$ lb. nearly.

2. Two forces, OX and OY, of 8 cwt. and 10 cwt. respectively, act at a point O in the gable of a textile machine, their lines of action being inclined to each other at 60°. Find the magnitude and direction of their resultant, and then resolve the resultant into two components, the lines of action of which are respectively parallel and perpendicular to OX. Determine the magnitude of these components. Find the result by graphical means.

Ans. 15·62 cwt. at 33° 40′ to OX.
13 cwt. parallel. 8·66 cwt. perpendicular.

3. A point on a very small warping mill travels 3 yards in one complete revolution; at the same time the heck-box with the threads descends 2 in.; what is the exact distance from the above point to the corresponding point in the next layer of yarn? The

point travels horizontally, the heck-box vertically, and the yarn obliquely.

Ans. 108·018 in., i.e., ·018 in. more than 3 yd.

Note: the difference is so small that it is never taken into account in warping calculations.

4. In a certain warehouse flax is unloaded from carts and raised inside the warehouse by means of an electric winch. The rope from the winch barrel passes over an overhead pulley; the part carrying the load hangs vertically; while the part leading from the pulley to the winch makes an angle of 30° with the vertical. If a bale of flax weigh 2 cwt., find the resultant force on the pulley stud. (See fig. 9, p. 21.) *Ans.* 432·7 lb.

5. A factory operative exerts a total force of 25 lb. in pulling a laden barrow along a level floor. The handle is attached to the barrow at a point 6 in. from the floor and the other end is in the operative's hand at a height of 30 in. from the floor. Find the effective force:—

(1) When the length of the handle is 3 ft., and
(2) When the length of the handle is 4 ft. 6 in.

What general inference may be drawn from these results?

Ans. 18·63 lb. 22·37 lb.

Chapter VI., pp. 56–63

1. State the Principle of the Triangle of Forces, and describe any experiment whereby its truth may be demonstrated.

2. A calender roller weighing 500 lb. is raised by a block and tackle in order to place it in its bearings in the gables of the machine. The roller hangs freely from the tackle 10 ft. below the point of suspension of the upper pulley block and 2 ft. horizontally from the bearing centres. What horizontal force will be required to push it forward into position in the bearings?

Ans. 100 lb.

3. To a scale of 1 in. to 5 ft. draw a triangle XYZ, making XY horizontal and equal to 10 ft., YZ 15 ft., and XZ 20 ft. XY represents the ground level, and YZ a pair of shear legs supported by a guy-rope XZ. Find the tension in the guy-rope, and the thrust in the shears when engaged in lifting the 5-ton fly-wheel of a mill engine.

Ans. Tension 2·55 tons. Thrust 7·10 tons.

4. X and Y are two points 3 ft. apart in the countermarch of the leaf of a power loom. Cords are fixed at these points and tied together to the upper part of a treadle hook Z, so that the angle XYZ is 40° and the angle ZYX is 45°. Find the tension in each cord if the total force used in shedding is 50 lb. Theoretically, the angles mentioned should be similar in magnitude; why? Solve the question by graphical means, or by mathematical means. *Ans.* XZ = 36 lb. YZ = 38·75 lb.

5. The post, jib, and tie of a factory courtyard crane measure respectively 15 ft., 35 ft., and 25 ft., while the plane of the backstays makes an angle of 45° with the ground. Find and name the stresses set up in each of the members by the lifting of a hydraulic press platen weighing 4 tons, hung from the point of the jib. Solve by graphical method.

Ans. Stress in jib	=	8·84	tons,	thrust or compression.
" tie	=	6·24	"	tension.
" post	=	2·68	"	compression.
" backstays	=	7·84	"	tension.

6. During erection, a textile machine weighing 20 cwt. is temporarily supported by two chains. One chain is attached to one of the roof principals and makes an angle of 20° with the vertical; the other chain is attached to a ring fixed in the wall and makes an angle of 20° with the horizontal. Find the pull in each of the chains.

Ans. Wall: 6·84 cwt. Roof: 18·8 cwt.

7. X is a point in a factory wall 6 ft. vertically above a second point Y. A triangular frame XYZ is swung on hinges at X and Y, the whole forming a wall crane of a simple type. If XZ = 10 ft., and YZ = 8 ft., find the stresses in XZ and YZ during the lifting of a bundling-press weighing 5 cwt., hanging from the point Z, and say which member is in compression and which in tension.

Ans. XZ: 6·67 cwt. tension. YZ: 8·33 cwt. compression.

Chapter VII, pp. 64–81

1. State exactly what is meant by the *moment* of a force; illustrate your answer by an explanatory sketch.

2. State the Principle of Moments, and describe an experiment which demonstrates its truth.

3. The diameter of a boiler safety-valve is 3 in. and its weight 4 lb.; the weight of the lever plus the connection to valve is 15 lb.; the distance from the fulcrum to the valve centre is 3 in., and to the centre of gravity of the lever $12\frac{1}{2}$ in. Find where a weight of 50 lb. must be placed on the lever so that steam will blow off at a pressure of 80 lb. per square inch. (See fig. 41.)

Ans. 29·94 in. from fulcrum.

4. In a certain type of jacquard loom the fulcrum of a lifting lever, 3 ft. long, is 1 ft. from the end. 1600 small weights (lingoes, 20 to the pound) have to be raised 4 in. by the short arm of the lever 120 times per minute. What is the force required at the end of the long arm, the number of ft.-lb. of work done per minute, and the H.P. required to perform this particular part of the weaving operation?

Ans. 40 lb. 3200 ft.-lb. (raising only).
·097 H.P. (raising only).

5. A rope is fixed to the low rail of a loom, passed over the beam-head or ruffle of the warp beam, and then attached to a weighted lever at a point 9 in. from the fulcrum. If a 28-lb. weight is placed 27 in. from the fulcrum, what is the pull on the rope? *Ans.* 84 lb.

6. If the diameter of the ruffle is 10 in., and the diameter over the warp beam of 1200 threads is 12 in., what is the tension on each warp thread? *Ans.* 0·93 oz.

7. Fig. 72 is a skeleton diagram of an Avery cloth tester; in which strips of cloth C, held between the grips G, are tested to destruction by applying a gradually increasing load in the form of lead shot automatically poured into the vessel V. The weight B balances the complete system when no cloth is being tested. In a certain test the gross load (vessel and shot) was 3 lb., 13 oz., 14 dr.; the vessel weighed 5 oz. 8 dr. Find the breaking stress in the cloth. *Ans.* 172 lb. approximately.

8. In a flax-spreader, pressure is applied to each pair of pressing-rollers by a system of compound levers. A weight of 15 lb. is hung at a distance of 27 in. from the fulcrum of the first or lower lever; a link is attached to a point 3 in. from the fulcrum of the first lever and 24 in. from the fulcrum of the second lever. The hanger from the pressing-roller journal is attached to the second lever at a point $3\frac{1}{2}$ in. from its fulcrum.

Find the pressure on the rollers, neglecting friction, weight of levers, &c. (See fig. 42.) *Ans.* 925·7 lb.

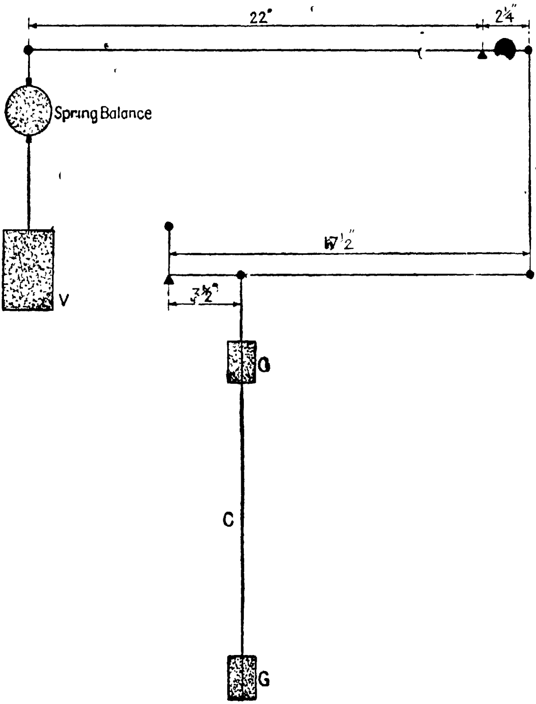

Fig. 72

9. In a certain lever safety-valve, such as is illustrated in fig. 41, the loading weight is 84 lb., the length of the lever from fulcrum to centre of weight is 28 in., fulcrum of lever to point of application, 3 in., fulcrum to centre of gravity of lever, 14 in., weight of valve, 6 lb., and weight of lever, 14 lb. If the blow-off pressure is 84 lb. per square inch, find the diameter of the valve.

Ans. 3·6 in.

10. In preliminary experiments performed to check the power required by a jacquard repeater (assuming that all punches acted simultaneously), an attempt was made to find the force required to drive a punch through a jacquard card. The average of 20 experiments with two different punches was 40 lb. for 10 holes. A skeleton diagram of the levers in the card cutter is given in fig. 73. If 40 lb. is the average force required to drive the punch through the card by a gradually applied load, find the corre-

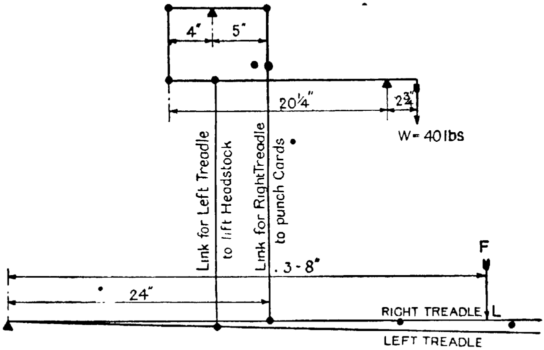

Fig. 73

sponding pressure which must be exerted at the end of the foot-lever L. *Ans.* 2·37 lb.

11. A plain loom is provided with treadles similar to that illustrated in fig. 37. The distance between F and W in one of the treadles is $28\frac{1}{4}$ in., and the distance between F and P is $21\frac{1}{2}$ in. If the shed is to be 4 in. deep, i.e., if the point at W has to move through 4 in., find the stroke or throw of the wyper, i.e. find the corresponding distance through which point P must be moved. *Ans.* 3·044 in.

12. In a certain spinning-frame, pressure is applied to a pair of rollers, each $\frac{5}{16}$ in. wide, by means of a lever and weight. The weight is 14 lb., and the lever weighs 2 lb. The weight acts at 12 in. from the fulcrum, and the point of application is $1\frac{1}{4}$ in. from the fulcrum. The centre of gravity of the lever is 5 in. from the fulcrum. Calculate the pressure on the rollers per inch width. *Ans.* 227·84 lb.

Chapter VIII, pp. 82–94

1. A uniform beam, supported at both ends 10 ft. apart, weighs 6 cwt., and supports a load of 2 cwt. at its centre. Find the reactions at the two supports. *Ans.* 4 cwt. at each.

2. A beam measures 8 ft. between its two supports, one at each end. It carries loads of 1, 2, and 3 cwt. at points 2, 4, and 6 ft. from its left-hand support. Determine the reactions at the two supports due to these loads alone.

Ans. Left-hand, $2\frac{1}{2}$ cwt.; right-hand, $3\frac{1}{2}$ cwt.

3. A uniform beam, 10 ft. long, and weighing 1000 lb., is supported at both ends. A load of 200 lb. is placed at a distance of 2 ft. from the left-hand end. Find the pressure and reaction at each point of support, and make a diagram of the arrangement to scale, marking on it the loads, distances, and reactions at each place. *Ans.* Left-hand, 660 lb.; right-hand, 540 lb.

4. A jacquard machine, weighing 3 cwt., is carried by two rolled-steel beams supported near the ends. The distance between the supports is 8 ft., while the centre of the machine is 3 ft. 6 in. from one end. Calculate the pressure in cwt. and in lb. on each support due to the weight of the jacquard.

Ans. Near the jacquard, $1\frac{11}{16}$ cwt. or 189 lb.
Other end, $1\frac{5}{16}$ cwt. or 147 lb.

5. Give an example from any textile machine with which you are acquainted of a couple producing rotation, and of the balancing of this couple by a second couple. Illustrate your answer with a descriptive sketch.

Chapter IX, pp. 95–102

1. State the Principle of Work, and explain the meaning of the terms "Lost Work", "Useful Work", and "Efficiency".

2. In a wheel and axle, which are respectively 3 ft. and 4 in. in diameter, a force of 50 lb. is found to be capable of raising a load of $382\frac{1}{2}$ lb. Find the velocity ratio, the mechanical advantage, and the efficiency at this load.

Ans. Velocity ratio = [illegible] Mechanical advantage = 7·65.
Efficiency = 0·85 or 85 per cent.

3. In a certain roving-frame the lifter or bobbin board, which weighs 1650 lb., is hung from chains attached to small pulleys

$4\frac{5}{8}$ in. effective diameter. These pulleys are keyed to shafts which carry at a different place larger pulleys, 7 in. effective diameter, on which are fixed chains to which 8 weights of 130 lb. each are attached. By how much does the lifter overbalance the weights? *Ans.* $175\frac{5}{8}$ lb.-in.

4. In a top roller of a plain tappet-loom, the bowl attached to the front leaf is $1\frac{3}{4}$ in. effective diameter, and that attached to the back leaf is $2\frac{1}{8}$ in. If the depth of the shed, measured at the front leaf, is $4\frac{1}{2}$ in., find the angle of rotation, and the depth of the shed at the back leaf. *Ans.* 294·7° using logs. 5·46 in.

5. The stroke of all the blades in a positive tappet-loom is the same, x inches, and the treadle-levers are fulcrumed at the back. If the front leaf travels 3 in., its cord attached 18 in. from the fulcrum of the jack-lever, and the cord from the treadle-lever also 18 in. from the opposite end of the jack-lever, find the distance from the fulcrum of the second jack-lever at its point of connection to the second treadle-lever if the shed at this point is $3\frac{1}{8}$ in. *Ans.* 17·28 in.

Chapter X, pp. 102–113

1. In Example 52, p. 113, the sign ≒ (*approximately* equal to) is used immediately before 30·56 rounds. Why is this sign used instead of the sign = (*equal* to)? Four lines below, the length of the warp is stated to be 305·6 yd.; what is the *exact* length? *Ans.* 305·607 yd.

2. Refer to fig. 72, p. 162, and calculate the mechanical advantage of the system of levers used in this form of cloth-testing machine. *Ans.* 48·8

3. A 3-and-4 rope tackle is employed to withdraw an air-pump during repairs being made to a mill engine. If the efficiency of the tackle is 80 per cent, and 4 men pull on the rope with an average force each of 56 lb., what resistance is being overcome? *Ans.* 1254·4 lb.

4. In experiments on a crane in which the velocity ratio is 40, the following results were found

Load lifted W	100 lb.	300 lb.	500 lb.	700 lb.
Force applied F	$8\frac{1}{2}$ lb.	17 lb.	$25\frac{1}{2}$ lb.	$34\frac{1}{4}$ lb.

Plot a curve on squared paper showing the relation between F and W on a W base. Show also an efficiency curve on the same base. Find from the curve the probable force required and the efficiency for a load of 4 cwt.

Ans. 23·4 lb. 47·84 per cent.

5. Write down definitions of the following terms: (1) Theoretical Mechanical Advantage; (2) Actual Mechanical Advantage; (3) Velocity Ratio; and (4) Efficiency. Assume reasonable figures for any simple lifting appliance, and work out examples for a particular case. How are these four quantities affected by an increase in the load?

Chapter XI, pp. 114–128

1. In a wheel and differential axle, the wheel is 24 in. diameter, the large pulley 6 in. diameter, and the small pulley 5 in. diameter; in each case the diameter is measured to the rope centres. Find the velocity ratio and the theoretical mechanical advantage. *Ans.* $\frac{1}{48}$. 48.

2. If the efficiency in Exercise 1 above is 65 per cent, what force must a factory operative exert in raising a sack of flour weighing 280 lb. from the ground to a starching-house store?

Ans. 8·975 lb.

3. In a Weston block the two pulleys are $5\frac{1}{2}$ in. and 5 in. diameter, measured to the chain centres. If the efficiency of the block is 45 per cent, what pull will be required to raise a beam of warp yarn weighing 480 lb.? *Ans.* 48·48 lb.

4. Describe how you could determine by experiment the velocity ratio and the actual mechanical advantage of a Weston differential pulley block.

5. Show by an example how a diagram may be used to correct the observation or measurement of an experiment. (See Example 56.)

6. In a certain spinning-frame, the heart cam, which actuates the traverse motion of the bobbin, has a throw of 6 in. It acts upon an anti-friction roller fitted in a lever and 13 in. from its fulcrum, while a stud is placed near the other end and $22\frac{1}{2}$ in. from the fulcrum. From the stud depends a rod terminating in a chain which passes partially round and is fixed to the surface of a large wheel. This large wheel is fixed to a shaft running

the length of the frame, and at regular intervals small circular bosses are keyed to this shaft. Chains are used to connect these bosses with brackets carrying the bobbin boards upon which the bobbins are placed. If the distance from fulcrum of lever to bowl centre is 13 in., from fulcrum to stud centre $22\frac{1}{2}$ in., and if the diameter of the large wheel is 9 in., and the effective diameter of the small bosses 5·2 in., calculate the traverse of the lever. *Ans.* 6 in.

Chapter XII, pp. 128–137

1. A barrel of cocoa-nut oil, weighing $2\frac{1}{2}$ cwt., is lowered into the basement of a factory down a smooth slide inclined at 45°. The barrel is lowered by two operatives, each of whom controls one rope of the parbuckle. Each rope thus passes from the man's hands partially round the barrel and back to the street level, where the end is fixed. Calculate the force exerted by each man in supporting the barrel at any point in its descent. *Ans.* 49·5 lb.

2. A tapered key is used to fix a pinion to a loom crank-shaft. If the taper is 1 in 96, and an estimated force of 100 lb. is used in driving the key to its final position, calculate the total resistance being overcome. *Ans.* 9600 lb.

3. In a certain roving-frame the differential wheel speed is controlled by a variable diameter (expanding or contracting) pulley. The sliding half of the pulley is moved into position by the turning of a triangular cam, see fig. 74. If it requires a force of 10 lb. to rotate the cam, find the resistance being overcome. *Ans.* $6\frac{1}{4}$ lb.

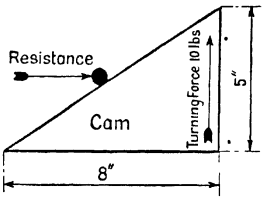

Fig. 74

4. A guide-bar, with guides &c., in a bobbin-winder weighs 50 lb. In order to equalize wear on the guides during its horizontal traverse, it is moved up an inclined plane having a slope of 1 in 3. Find the total horizontal force required if 10 per cent of the total is used up in overcoming friction. *Ans.* 19·62 lb.

Chapter XIII, pp. 138–142

1. Suggest any means of determining experimentally the velocity ratio and the actual mechanical advantage of an ordinary screw-jack (bottle-jack).

2. The pitch of the screw in a screw-jack is $\frac{3}{8}$ in., and it is turned by a tommy-bar 20 in. long; compare the speed of the end of the tommy-bar with the speed at which the load is lifted Use 3·14 for π. *Ans.* 334·9 to 1.

3. In experimenting with a screw-jack it is found that when the force applied moves through a distance of $31\frac{1}{2}$ in., the load is raised $\frac{1}{2}$ in., and that it requires a force of $2\frac{3}{4}$ lb. to raise a load of 61 lb. From this data find—(1) the velocity ratio, (2) the theoretical mechanical advantage, (3) the actual mechanical advantage, and (4) the efficiency at this load.
Ans. (1) $\frac{1}{63}$; (2) 63; (3) 22·18; (4) 0·352.

4. In a certain type of hemp softener, pressure is applied to the upper roller by means of two hand-wheels, 15 in. diameter, each controlling a screw $1\frac{1}{2}$ in. diameter, single square-threaded, $\frac{1}{4}$-in. pitch. Find the pressure exerted on the upper roller when a turning force of 30 lb. is applied at each hand-wheel. Efficiency of the screw is 45 per cent. *Ans.* 5086·8 lb.

5. In an ordinary bench-vice, the pitch of the screw is $\frac{1}{4}$ in., the distance from the fulcrum to centre line of screw is 12 in., the distance from the fulcrum to the centre of the object gripped between the jaws is 16 in., and the effective length of the handle is 14 in. Find the gripping force produced in the jaws by a pull of 20 lb. at the end of the handle; the efficiency of the screw is 35 per cent. Take $\pi = \frac{22}{7}$. *Ans.* 1848 lb.

6. A loom is held to the floor by 4 bolts, $\frac{7}{8}$ in. diameter. Find the gripping force exerted by each nut when they are screwed up with a force of 28 lb. at the end of a spanner 15 in. long. The screw has 9 threads per inch, and the efficiency may be taken at $\frac{1}{3}$. *Ans.* 7920 lb.

7. Generally speaking, all machines should have a high efficiency. Give reasons, with illustrations from your own experience, when it is advantageous for such a simple machine as a screw to have a low efficiency.

Chapter XIV, pp. 142–154

1. A 6-ft.-diameter pulley, running at 120 r.p.m., drives a second pulley 30 in. diameter on a factory counter-shaft by means of a belt. If there is 5 per cent slip on the belt, what is the speed of the counter-shaft? If the pulley transmits 10 H.P., what will be the driving pull in the belt? *Ans.* 273·6 r.p.m. 153·5 lb.

2. The pull on the driving side of a belt is 200 lb., and on the following side 60 lb., while the belt runs at 990 ft. per minute. Find the number of units of work done per minute, the H.P. transmitted, and the width of the belt at 40 lb. pull per inch of width. *Ans.* 138,600 ft.-lb. 4·2 H.P. 5 in.

3. A belt transmits 60 H.P. to a factory dynamo pulley 16 in. diameter running at 480 r.p.m. Find the driving pull in lb. If the ratio of the tension in the tight and slack sides is 10:3, find the tension in the tight side of the belt in lb. Use $\pi = \frac{22}{7}$.
Ans. 984·4 lb. 1406·25 lb.

4. A factory main-shaft, running at 100 r.p.m., drives a machine taking 6 H.P., by means of a belt running on a 5-ft.-diameter pulley. The belt used can stand a working pull of 42 lb. per inch width, and the tensions on the tight and slack sides are as 10 to 4. Find a suitable width for the belt. *Ans.* 5 in.

5. A $1\frac{3}{4}$-in. rope transmits 47 H.P. to a rope-pulley 8 ft. diameter. If the rope speed is to be 4800 ft. per minute, find the pulley speed in r.p.m. What must be the difference in the tensile forces in the rope on the two sides of the pulley?
Ans. 191·08 r.p.m. $323\frac{1}{8}$ lb.

6. Two rollers in a dust-shaker must revolve in the same direction, and the speed of the first be $2\frac{1}{2}$ times that of the second. If the centres are 11 in. apart, and the gears connecting them are No. 6 Diametral Pitch, find one set of suitable numbers of teeth for the driving and driven wheels.
Ans. 16 teeth in first and 40 teeth in second. Any reasonable number, say 40, in idle wheel.

7. A card-room shaft makes 180 r.p.m. A drum on the shaft drives a card-pulley, 24 in. diameter, at a speed of 200 r.p.m. What should be the diameter of the drum?
Ans. 26·67 in.

8. In a ring spinning-frame the ratio of the tin cylinder to the spindle wharve is 10 to 1; the driving pulley on the shaft of the tin cylinder is 14 in. diameter, and the spinning-room shaft runs at 190 r.p.m. If the spindle speed is to be 6500 r.p.m., find the diameter of the driving drum. *Ans.* Approx. 48 in.

9. If a plain loom has on the end of the crank-shaft a wheel with 40 teeth, how many teeth should there be in the wheel of the tappet-shaft? The latter revolves at half the speed of the former. *Ans.* 80 teeth.

10. Refer to Exercise 9 above. If the driving pulley or drum on the line-shaft is 14 in. diameter and runs at 150 r.p.m., what will be the speed of the tappet-shaft, given that the pulley on the crank-shaft is $17\frac{1}{2}$ in. diameter? *Ans.* 60 r p.m.

11. Hemp ropes are employed to transmit power from an engine fly-wheel to the various floors of a spinning-mill. The maximum tension in a rope is three times the minimum tension, and the greatest stress in a rope is not to exceed 530 lb. How many ropes will be required to transmit 415 H.P. if the rope speed is to be 70 ft. per second? *Ans.* 9 ropes.

12. A jute-softener is driven through a 48-in.-diameter pulley by two belts running one on top of the other, and each $\frac{3}{8}$ in. thick. The speed of the middle plane of the inner belt is 1700 ft. per minute; find by how much the outer belt gains on the inner belt each minute. *Ans.* 43·7 ft.

13. In a certain roving-frame the main-shaft runs at 400 r.p.m. and carries a 44 pinion. This pinion gears with a single intermediate of 60 teeth, which in turn gears with the 82 wheel of a double intermediate, the pinion of which has 45 teeth. The latter pinion gears with a 90 on the drawing-roller. Find the speed of the drawing-roller in r.p.m., and say in what direction it rotates with reference to the main-shaft.

Ans. 107·3 r.p.m. Opposite direction to main-shaft.

MISCELLANEOUS EXERCISES

1. A 10-H.P. motor weighing 10 cwt., and used in the group-driving of textile machinery, is supported on the upper surface of a wall-bracket, which itself weighs $3\frac{1}{3}$ cwt. If the centre of gravity of the motor is 2 ft. from the wall, and the centre of gravity of the bracket is 18 in. from the wall, find the combined moment exerted by these loads in ton-inches. *Ans.* 15 ton-in.

2. A hand jacquard loom, shown diagrammatically at A in fig. 75, contains 400 hooks which control 1600 lingoes B and harness cords C. The combined weight of one lingoe and its cord is 1 oz. The lifting apparatus consists of two levers, DE and HG, connected by a rope F. What pressure in pounds must be exerted by the foot at K in order to lift half the threads? *Ans.* $37\frac{1}{2}$ lb.

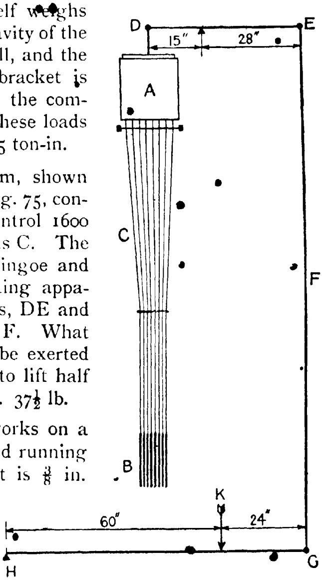

Fig 75.

3. A belt, 8 in. wide, works on a pulley 3 ft. in diameter and running at 120 r.p.m. If the belt is $\frac{3}{8}$ in. thick, and the driving pull 280 lb. per square inch of section, find the H.P. transmitted. (Use $\pi = \frac{22}{7}$.)
Ans. 28·8 H.P.

4. A winding-room shaft is driven by a drum of 30 in. diameter, and runs at 180 r.p.m. If the power transmitted is 30 H.P., and the driving pull in the belt is to be 70 lb. per inch width, find the width of the belt. (Use $\pi = \frac{22}{7}$.)

5. One end of a rope is attached to the movable hook of a spring balance, and the opposite end of the spring balance is fixed to some stationary part of a machine. The rope is wound about the fast pulley, passed over a guide pulley, and a pan attached to the end for weights. When 80 lb. in weight have been added to the pan, the pulley commences to move, and the spring balance registers 10 lb. If the pulley is 16 in. diameter, and the revolutions per minute intended to be 360, what H.P. will be required to drive the machine? (The 80 lb. may be taken as including the weight of the pan, and $\pi = \frac{22}{7}$.)

Ans. 3·2 H.P.

6. A 5-in. belt, $\frac{1}{4}$ in. thick, works on the pulley of a machine and runs at 1600 r.p.m. What H.P. will be transmitted if the working pull in the belt is 330 lb. per square inch of section?

Ans. 20 H.P.

7. A shuttle and cop weigh $2\frac{1}{2}$ lb. The loaded shuttle is placed in the shed, a string attached to it, and the other end of the string passed over a pulley. If it requires weights to the value of 14 oz. to move the shuttle on the warp threads, and if $\frac{P}{W}$ is the coefficient of friction, find the value of this coefficient for the above case. *Ans.* 0·35.

8. A 5-wheel uptake motion in a power loom has three driven wheels and two driver wheels. The first driven wheel is a ratchet of 44 teeth, which is moved one tooth for every pick of the loom; the last driven wheel of 120 teeth is on the roller which draws forward the cloth. If the third driven wheel has 100 teeth, and the two driver wheels have 30 and 40 teeth respectively, how many shots of weft will be inserted in the cloth during one complete revolution of the take-up roller? *Ans.* 440 shots.

9. If the take-up roller in Example 8 above is 5 in. dia., how many shots will be inserted in each inch of the cloth? (Use $\pi = \frac{22}{7}$.) *Ans.* 28 shots per inch.

10. The traverse lever in a certain roll winder stands vertically when in mid-stroke; the stud on which it oscillates is at the lower end. The cam which causes the lever to reciprocate acts on an anti-friction bowl, and the stud of the bowl is fixed into a vertical slot of the lever, the centre of the slot being 15 in. from the fulcrum of the lever; the cam has a throw of $3\frac{1}{4}$ in. The traverse bar carrying the yarn guides is actuated directly from a stud fixed in a second vertical slot at the upper end of the lever;

the centre of this slot is $38\frac{1}{2}$ in. from the fulcrum of the lever. The upper slot allows an adjustment of $\frac{3}{4}$ in. up and $\frac{3}{4}$ in. down; the lower slot allows an adjustment of $3\frac{1}{2}$ in. up and $3\frac{1}{2}$ in. down. Draw a skeleton diagram of the arrangement described, and find the maximum and minimum travel of the yarn guides.

Ans. 11·09 in. max. and 6·63 in. min.

11. In the negative let-off motion of a power loom, an upward pull of 20 lb., applied to a handle at one side of the loom, acts at 12 in. from the fulcrum, and is transmitted by a chain, passing over a block $4\frac{1}{2}$ in. from the fulcrum, to one end of a 2-armed lever passing from the front of the loom to the back. The distance from this end of the lever to the fulcrum is 4 ft. 4 in., and from the fulcrum to the line of application of the pressure on the warp beam, 6 in. Find the pressure on the beam due to this pull and leverage. *Ans.* 462·2 lb.

12. In a system of compound levers, the lengths of the arms of the three levers are as follows:—

1st lever—3 in. and 24 in.
2nd ,, —$2\frac{1}{2}$ in. and 25 in.
3rd ,, —2 in. and 26 in.

A force of 60 lb. is caused to act on the long arm of the 3rd lever. Find the resultant force on the short arm of the 1st lever, when the system is arranged to give the maximum theoretical mechanical advantage. *Ans.* 62,400 lb.

13. The following particulars are taken from a 96-in. 5-bowl calender, illustrated diagrammatically in fig. 76. Top roller A, 20 in. dia., with handwheels, screws, blocks, and arbors, weighs 60 cwt.; upper paper roller B, 27 in. dia., with gear, 48 cwt.; driving or steam roller C, $13\frac{1}{2}$ in. dia., with gear, 38 cwt.; lower paper roller D, 27 in. dia., with gear, 48 cwt. Additional pressure is obtained by a pair of levers, of which one only is shown, F, weighing $16\frac{1}{4}$ cwt. At the end of each lever is fixed a hanging rod G terminating in a rack H, which gears with a rack pinion J, 7 in. pitch dia. The racks H and rods G weigh $3\frac{1}{4}$ cwt Each pinion J is compounded with a pulley K, 30 in. dia., from the periphery of which depends a hanger L, carrying 10 weights W of 50 lb. each. Find:—

(1) The dead-weight pressure on the bottom bowl E.
(2) The pressure due to the weight of the levers F alone.
(3) The pressure due to the weight of the racks H, hangers L, pins, &c.

(4) The pressure due to the rack pinions J, pulley K, and weights L.

(5) The total pressure on the bottom roller E of the calender.

Ans. (1) 194 cwt. (2) 65 cwt. (3) $35\frac{3}{4}$ cwt.
(4) 210·46 cwt. (5) 505·21 cwt., say $25\frac{1}{4}$ tons.

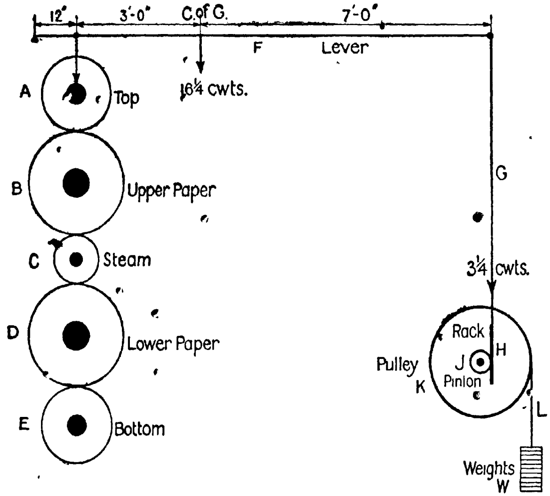

Fig 76

14. A centrifugal pump in a large factory condensing plant has to raise 6 c. ft. of water per second to a height of 7 ft. What H.P. will be required if the efficiency of the pump is 45 per cent?
Ans. 10·606 H.P.

15. Tests on a lifting winch gave the following results:—

Force applied in pounds ...	5	10	15	20	25	30
Weight raised in pounds ...	100	195	290	380	460	540

Plot a graph of these values and find from it the approximate force necessary for a load of 500 lb. *Ans.* $27\frac{1}{2}$ lb.

16. Calculate the horse-power required to pump 5000 gall. of water per minute from a well the surface of which is 40 ft. deep. Assume that the efficiency of the machinery employed is 60 per cent. *Ans.* 101·01 H.P.

17. A rod 10 ft. long is supported at each end. At distances of 2, 4, 5, and 7 ft. respectively from the left-hand end, loads of 5, 4, 3, and 2 cwt. are supported. Find the reactions at the two supports due to these loads.
Ans. Right-hand: $5\frac{1}{2}$ cwt. Left-hand: $8\frac{1}{2}$ cwt.

18. Two toothed wheels, mounted on parallel shafts, are to gear with each other, and their speeds of rotation are to be as 2 is to 1. If the distance between the centres of the two shafts is 12 in., and the pitch of the teeth No. 6 Diametral, find the number of teeth in each wheel. *Ans.* 48 and 96.

19. A loaded truck weighs 10 cwt. Calculate the effort in pounds exerted by each of two textile operatives in pulling it up an incline of 1 in 20, when the road resistance on the level is 40 lb. per ton. *Ans.* 38 lb.

20. Sketch any textile mechanism in which a screw is used to obtain mechanical advantage. In a certain piece of mechanism of this type, a force of 42 lb. is applied at the end of a 5-ft. lever to a screw $\frac{3}{4}$-in. pitch and 30 per cent efficiency. Find the resistance overcome in hundredweights. (Use $\pi = \frac{22}{7}$.)
Ans. $56\frac{4}{7}$ cwt.

21. A lathe in a textile machine-shop is driven by a 5-in. belt running at 1500 ft. per minute, and absorbs 5 H.P. If the tensions in the tight and slack sides are as 10 is to 3, find the greatest tension in the belt per inch of width. *Ans.* 31·43 lb.

22. A set of pulley blocks has 3 pulleys in the lower block and 4 pulleys in the upper block. What is the theoretical mechanical advantage of the system? If its efficiency is 50 per cent, what weight would be lifted by a pull of 40 lb. on the rope?
Ans. T.M.A. = 7. 154 lb.

23. A drawing-frame has a 16-in. dia. pulley which normally rotates at 250 r.p.m. It is found that a tangential pull of 63 lb. exerted at the rim of the pulley is just sufficient to start the machine. Calculate the approximate horse-power which will be used. (Use $\pi = \frac{22}{7}$.) *Ans.* 2 H.P.

24. The constant number of teeth in two wheels for driving a

supplementary shedding wyper or tappet in a twill loom is 90; find the numbers of teeth for the following:—

(*a*) A 3-pick to round weave.
(*b*) A 4-pick ,, ,,

Explain why the above so-called constant number should be one tooth more for a 5-pick to round weave.

Ans. (*a*) 36 and 54. (*b*) 30 and 60.

USEFUL CONSTANTS

1 c. ft. of water	= 6¼ gall.	= 62·5 lb.
1 gall. of water	= ·1605 c. ft.	= 10 lb.
1 in.	= ·083 ft.	= 2·54 cm.
1 sq. in.	= ·0069 sq. ft.	= 6·45 sq. cm.
1 c. in.	= ·000572 c. ft.	= 16·39 c. c.
1 metre	= 3·28 ft.	= 39·37 in.
1 sq. m.	= 10·76 sq. ft.	= 1550 sq. in.
1 c. m.	= 1·308 c. yd.	= 61,025 c. in.
1 fathom	= 6 ft.	= 2 yd.
1 chain	= 66 ft.	= 22 yd.
1 mile	= 80 chains	= 5280 ft.
1 naut	= 1 nautical mile	= 6080 ft.
1 knot	= 1 naut per hour	= 6080 ft. per hour.
1 litre	= 1000 c. c.	= 1·762 pt.
1 gall.	= ·1605 c. ft.	= 4·541 litres.
1 bus.	= 8 gall.	= 1·2837 c. ft.
1 lb. avoirdupois	= 7000 gr.	= 453·6 gm.
1 Kgm.	= 15,435 gr.	= 2·205 lb.
1 oz.	= 437·5 gr.	= 28·35 gm.

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